Lecture - 10-9-14 Professor Shabanskaya Calculus 1760-002

Quiz 2 on Friday

Covers: Summation & Evaluation of Sums

Area Approximation

The Definate integral

2 problems, Ten minutes at class end.

<u>Section 6.1</u>: The definate integral

Definition: Let f be a function ~~#~~ defined on $[a,b]$. Then the definate intergral of f on $[a,b]$ denoted by:

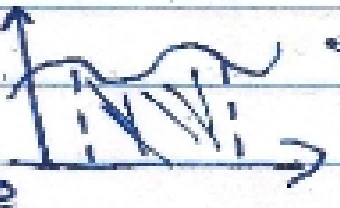
$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x, \text{ where}$$ provided this limit exists

$\Delta x = \frac{b-a}{n}$ - the length of each of the n equal lengths sub intervals $[x_0, x_1], [x_1, x_2], [x_2, x_3], \cdots, [x_{i-1}, x_i], \cdots, [x_{n-1}, x_n]$ ($x_0 = a$, $x_n = b$) and the points x_i^* are in $[x_{i-1}, x_i]$, $i = 1, 2, 3, \cdots, n-1, n$

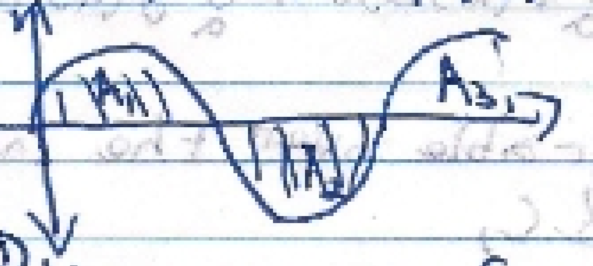
<u>Geometric Properties</u>:

1) If f is ~~in~~ integrable on $[a,b]$, & $f(x) \geq 0$ for $a \leq x \leq b$, then

$\int_a^b f(x)\,dx \geq 0$ & it is the area of the region below $f(x)$ for $a \leq x \leq b$



2) $\int_a^b f(x)\,dx$ = f is integrable on $[a,b]$ = sum of the areas below $f(x)$ - sum of the areas above $f(x)$



$\underbrace{A_1 + A_3}_{\text{sum of areas}} - \underbrace{A_2}_{\text{area}}$

<u>Ex</u>: ① Use an area formula to find the value of the following: $\int_2^{10} (\tfrac{1}{2}x - 4)\,dx$ by interpreting it as the (signed) area. ~~under the graph of~~