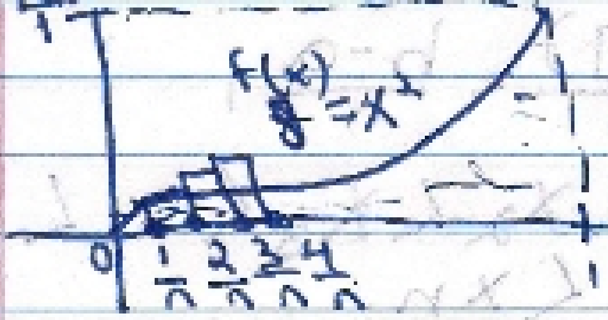
3.) The net area or the signed area mean you are just asked to calculate the definate integral $\int_a^b f(x)\,dx$. (the answer could be negative)

Ex: Calculate the area below $y=x^2$ on $[0,1]$.



1.) Divide $[0,1]$ into n equal parts with the lengths of each equal part to be $\Delta x = \frac{\ell([0,1])}{n} = \frac{1}{n}$

2) We have n subintervals of $[0,1]$:
$[0, 1/n], [1/n, 2/n], [2/n, 3/n], \cdots, [\frac{i-1}{n}, i/n], \cdots$
$[\frac{n-1}{n}, \frac{n}{n}]$ $(n/n = 1)$

3.) Right end points: $1/n, 2/n, 3/n, \cdots, i/n, n/n$
$x_i = x_0 + i \cdot \frac{1}{n}$ $\left(\frac{\ell([0,1])}{n}\right)$, $i = 1, 2, 3, \cdots, n$, because $x_0 = 0$
$\Rightarrow x_i = i \cdot 1/n = i/n$

4.) $\underbrace{\sum_{i=1}^{n} f(x_i)\Delta x}_{\text{Riemann sum}} \underset{x_i = i/n}{=} \sum_{i=1}^{n} f(i/n)\Delta x \underset{\Delta x = 1/n}{=}$

$= \sum_{i=1}^{n} (i/n)^2 \underbrace{(1/n)}_{\text{does not depend on } i} = 1/n \sum_{i=1}^{n} \underbrace{\left(\frac{i}{n}\right)^2}_{= i^2/n^2} = 1/n \cdot 1/n^2 \sum_{i=1}^{n} i^2 =$

$= 1/n^3 \sum_{i=1}^{n} i^2 = 1/n^3 \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6n^2}$

$A_{y=x^2}$ (area below $y = x^2$ on $[0,1]$)

$= \int_0^1 x^2\,dx = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} = \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2} =$

$= \lim_{n\to\infty} \left(1/3 + 1/2n + 1/6n^2\right) = 1/3 + 0 + 0 = \boxed{1/3}$