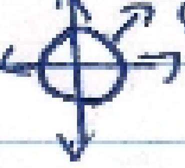
$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx - \int_b^c f(x)\,dx$

6.) $f(x) \geq 0$ on $[a,b] \Rightarrow \int_a^b f(x)\,dx \geq 0$

7.) $f(x) \leq g(x)$, $a \leq x \leq b$ $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$
$(g(x) - f(x) \geq 0)$

8.) If $m \leq f(x) \leq M$, $a \leq x \leq b$ then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

P. 293 #78 Verify $1/2 \leq \int_0^1 \sqrt{1-x^2}\,dx \leq 1$ without evaluating the integrals.

Solution:  $f(x) = \sqrt{1-x^2} \geq 0$ on $[0,1]$

$\sqrt{1-x^2} \geq 0 \Rightarrow \int_0^1 \sqrt{1-x^2}\,dx \geq 0 \Rightarrow$ it is the →
$0 \leq x \leq 1$

→ area below $f(x) = \sqrt{1-x^2}$ on $[0,1] \Rightarrow$

$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4} \geq 1/2$

Notice $\sqrt{1-x^2} \leq 1$ where $0 \leq x \leq 1$

$\int_0^1 \sqrt{1-x^2}\,dx \leq \int_0^1 1\,dx \leq 1$