Lecture: 1-10-14 Professor Shabanskaya Calculus 1760-002

## Section 7.3: Rational Functions & Partial Fractions

$\int \frac{P(x)}{Q(x)}\,dx$ We have the following degree $(P(x)) <$ degree $(Q(x))$ in $\frac{P(x)}{Q(x)}$

1.) $\frac{P(x)}{a(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)} = \frac{1}{a}\left[\frac{A_1}{x-x_1}+\frac{A_2}{x-x_2}+\frac{A_3}{x-x_3}+\dots+\frac{A_n}{x-x_n}\right]$,

$A_1, A_2, A_3, \dots, A_n$ are constants (so are $x_1 - x_n$)

2.) $\frac{P(x)}{a(x-b)^n} = \frac{1}{a}\left[\frac{B_1}{x-b}+\frac{B_2}{(x-b)^2}+\frac{B_3}{(x-b)^3}+\dots+\frac{B_n}{(x-b)^n}\right]$

$B_1, B_2, B_3, B_n$ are constants

3.) $\frac{P(x)}{(ax^2+bx+c)^n} = \frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\frac{A_3x+B_3}{(ax^2+bx+c)^3}+\dots+\frac{A_nx+B_n}{(ax^2+bx+c)^n}]$

irreducable polynomial, can not be factored more

if $D = b^2-4ac < 0$ ($D$ = discriminant), the polynomial $ax^2+bx+c$ is irreducable

4.) $\frac{P(x)}{(a_1x^2+b_1x+c_1)(a_2x^2+b_2x+c_2)} = \frac{C_1x+B_1}{a_1x^2+b_1+c_1}+\frac{C_2x+B_2}{a_2x^2+b_2x+c_2}$

both irreducable

Ex: p 351 use partial fraction decomposition to evaluate the ~~interval~~ integrals

#33 $\int\frac{x^2+1}{x^2+3x+2}\,dx$ = (applied long division) $\int 1\,dx + \int\frac{-3x-1}{x^2+3x+2}\,dx =$

$x + \int\frac{-3x-1}{x^2+3x+2}\,dx = x+\int \frac{A_1}{(x+1)}+\frac{A_2}{(x+2)}\,dx = x+\int\frac{2}{(x+1)}+\frac{5}{(x+2)}\,dx =$

$\frac{-3x-1}{x^2+3x+2} = \frac{-3x-1}{(x+1)(x+2)} = \frac{A_1}{(x+1)}+\frac{A_2}{(x+2)} = \frac{A_1(x+2)+A_2(x+1)}{(x+1)(x+2)}$ (compare what you have on the left & right)

$= A_1(x+2)+A_2(x+1) = -3x-1$ (plug $x=-1$ in) then (plug $x=-2$)

a. $A_1(-1+2) = -3(-1)-1 = \boxed{A_1(1)=2}$

b. $A_2(-2+1) = -3(-2)-1 = A_2(-1)=5 = \boxed{A_2=-5}$

$x+\int\frac{2}{(x+1)}+\int\frac{5}{(x+2)}\,dx = 2\int\frac{1}{x+1}\,dx = 2\int\frac{1}{u}\,du = 2\ln|u|+C = 2\ln|x+1|+C =$

Set $u=x+1$ $du=1\,dx=dx$

$= x+2\ln|x+1|-5\ln|x+2|+C$

Ex: #20 $\int\frac{x^3-3x^2+x-6}{(x^2+2)(x^2+1)}\,dx = \int\frac{A_1x+B_1}{x^2+2}\,dx+\int\frac{A_2x+B_2}{x^2+1}\,dx$

1.) check if degree $\underbrace{(x^3-3x^2+x-6)}_{3}$ < degree $\underbrace{(x^2+2)(x^2+1)}_{4}$

2. $\frac{x^3-3x^2+x-6}{(x^2+2)(x^2+1)}$ (both are irreducable fractions)

$= \frac{A_1x+B_1}{x^2+2}+\frac{A_2x+B_2}{x^2+1} = \frac{(A_1x+B_1)(x^2+1)+(A_2x+B_2)(x^2+2)}{(x^2+1)(x^2+2)}$ (numerators are same)

$\frac{A_1x^3+A_1x+B_1x^2+B_1+A_2x^3+2A_2x+B_2x^2+2B_2}{(x^2+1)(x^2+2)} = x^3-3x^2+x-6$

$(A_1+A_2)x^3+(B_1+B_2)x^2+(A_1+2A_2)x+(B_1+2B_2) = x^3-3x^2+x-6$

$A_1+A_2=1,\ B_1+B_2=-3,\ A_1+2A_2=1,\ B_1+2B_2=-6$

Homework 351 # 14, 17, 19, 21