Lecture: 8-5-14 Professor Shabanskaya Calculus 1760-002

Section 6.1: The definate intergral

Definition: Let f be a function defined on the closed interval [a, b]. Divide the interval [a, b] into n equal parts with the length of each part

$$\Delta x = \overset{\ell}{\text{length}}\,\frac{([a,b])}{n} = \frac{b-a}{n},$$

such that $a = x_0 < x_1 < x_2 < x_3 < \cdots < x_{i-1} < x_i < \cdots < x_{n-1} < x_n$.

We have n subintervals of [a, b].

$[\underset{a}{x_0}, x_1], [x_1, x_2], [x_2, x_3], \ldots [x_{i-1}, x_i], \ldots [x_{n-1}, \underset{b}{x_n}]$

We choose the points $x_i^* \in [x_{i-1}, x_i]$, $i = 1, 2, 3, \cdots n-1, n$ $(i = \overline{1, n})$

Then the definate integral of f(x) on the closed interval [a, b], denoted as:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$ (provided that this limit exists)

This is called Riemann sum

Besides if this limit exists f is called integrable on [a, b]

Remarks:

1.) Most of the time we choose x_i^* to be

a) left endpoints of subintervals: $x_i = x_0 + i\frac{b-a}{n}$, $i = \overline{0, n-1}$

b) Right endpoints: $x_i = x_0 + i\frac{b-a}{n}$, $i = \overline{1, n}$ $(i = 1, 2, 3, \cdots, n)$

c.) midpoints: $\overline{x_i} = \frac{x_{i-1} + x_i}{2}$, $i = \overline{1, n}$

2.) If f is integrable on [a, b] and besides $f(x) \geq 0$, then $\int_a^b f(x)\,dx \geq 0$ and it is the area below f(x) on the closed interval [a, b].

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x =$