$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx - \int_b^c f(x)\,dx$

6.) $f(x) \ge 0$ on $[a,b] \Rightarrow \int_a^b f(x)\,dx \ge 0$

7.) $f(x) \le g(x)$, $a \le x \le b$ $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$
$(g(x) - f(x) \ge 0)$

8.) If $m \le f(x) \le M$, $a \le x \le b$ then $m(b-a) \le$
$\int_a^b f(x)\,dx \le M(b-a)$

P. 293 #78 Verify $1/2 \le \int_0^1 \sqrt{1-x^2}\,dx \le 1$
without evaluating the integrals.

Solution:  $f(x) = \sqrt{1-x^2} \ge 0$ on $[0,1]$

$\sqrt{1-x^2} \ge 0 \Rightarrow \int_0^1 \sqrt{1-x^2}\,dx \ge 0 \Rightarrow$ it is the →
$0 \le x \le 1$

→ area below $f(x) = \sqrt{1-x^2}$ on $[0,1] \Rightarrow$
$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4} \ge 1/2$

Notice $\sqrt{1-x^2} \le 1$ where $0 \le x \le 1$
$\int_0^1 \sqrt{1-x^2}\,dx \le \int_0^1 1\,dx = 1$

Lecture: 12-~~8~~9-14 Professor Shabanskaya Calculus 1760-002
Test moved to September 26th

~~6.2~~ Section 6.2: The Fundamental Theorem of Calculus

Theorem: (Fundamental Theorem of Calculus)
Let f be a continuous function on $[a,b]$.
Then:

Part 1) Let $F(x) = \int_a^x f(u)\,du$ ($a \le x \le b$). Then F is continuous on $[a,b]$ & differentiable on $(a,b)$, and →