Lecture: 2-28-14 Professor Shabanskaya Calculus 1760-002

Section 5.8 Antiderivatives

Definition: Let f be a function defined on some interval I. Then the function F is an antiderivative of f if $\frac{d}{dx}F(x) = f(x)$ for x in I.

Remark: $F(x)+C$, (C is a constant) is the set of all antiderivatives of $f(x)$ or the general antiderivative of $f(x)$. Besides $F(x)$ could be called a particular antiderivative of $f(x)$.

Ex: Find all antiderivatives of $f(x) = x^{-5}$

Solution: 1.) Find a particular antiderivative
2.) Add C to the result. Hint: Power rule $\frac{d}{dx}(x^r) = r \cdot x^{r-1}$

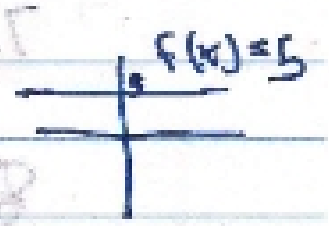
1.) ~~$\frac{1}{-4}x^{-4}$~~ Suggestion: $-\frac{1}{4}x^{-4}$ is a particular antiderivative because $\frac{d}{dx}(-\frac{1}{4}x^{-4}) = -\frac{1}{4}\frac{d}{dx}(x^{-4}) = -\frac{1}{4}(-4)x^{-5}) = -\frac{1}{4}\cdot -4 = 1$
$1x^{-5} = \boxed{x^{-5}}$

2.) $\boxed{-\frac{1}{4}x^{-4}+C}$ C is a constant for all antiderivatives of $f(x) = x$

Indefinite Integral

The set of all antiderivatives of the function f is called the indefinite Integral of f with respect to x and denoted by: $\int f(x)\underbrace{dx}_{\text{differential}} = F(x)+C$, C is a constant.

Ex: ① Find the indefinite integral $\int 5\,dx$. ($f(x)=5$)
(All antiderivatives of function $f(x)=5$ / the general antiderivative)



Solution: 1.) Find particular antiderivative $f(x) = 5$
2.) add C to result
1.) Suggestion: $5x$ is a particular antiderivative because $\frac{d}{dx}5x = 5\frac{d}{dx}x$ ~~$\frac{d}{dx}5$~~,
2.) $\boxed{5x+C}$ C is a constant for all antiderivatives.
$\Rightarrow \int 5\,dx = 5x+C$, C is a constant