

where I is an interval, then $\int_a^b F(g(x))g'(x)dx = \int_{g(a)}^{g(b)} F(u)du$

Ex: Use the Substitution method to evaluate the definite integral.

$$1.) \int_0^1 6x^5 (1+x^6)^5 dx = \int_1^2 (u)^5 \frac{du}{6x^5} = \int_1^2 u^5 du = \Rightarrow$$

$$1.) \text{ Set } u = 1+x^6, \quad x=0 \Rightarrow u=1, \quad x=1 \Rightarrow u=2$$

$$2.) \frac{du}{6x^5} = 6x^5 dx = du \Rightarrow \frac{du}{6x^5} = dx$$

$$3.) \text{ If } x=0, \text{ use } u=1+x^6, \quad u=1+0^6 = 1$$

$$\text{If } x=1, \quad u=1+x^6, \quad u=1+1^6 = 2$$

$$\Rightarrow = \frac{u^6}{6} \Big|_1^2 = \frac{2^6}{6} - \frac{1^6}{6} = \frac{64}{6} - \frac{1}{6} = \frac{63}{6}$$

Lecture: 22-9-14 Calculus 1160-002 Professor Shabanokaya

This is not on this coming test.

Section 7.2: Integration by Parts & Practicing

Integration

Integration by Parts: Let $u(x)$ & $v(x)$ be differentiable functions. Then $\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$.

$$\text{or } \int u dv = uv - \int v du \quad \begin{matrix} \rightarrow dv = v'(x) dx \\ du = u'(x) dx \end{matrix}$$

Proof: Consider $(u(x)v(x))' = \frac{d}{dx}(u(x)v(x)) =$

$$u(x)v'(x) + u'(x)v(x). \int (u(x)v(x))' dx =$$

$$= \int u'(x)v(x) dx + \int u(x)v'(x) dx$$

$$u(x)v(x) = \int u'(x)v(x) dx + \int u(x)v'(x) dx$$

$$\Rightarrow \int u(x)v'(x) dx = u(x)v(x) - \int u'(x)v(x) dx$$

We have $\int u(x)v'(x) dx = u(x)v(x) - \int u'(x)v(x) dx$

Ex: Evaluate using integration by parts or substitution.

Check by differentiating.

$$a.) \int 8x e^{9x} dx = 8 \int x e^{9x} dx = 8x \cdot \frac{1}{9} e^{9x} - \int 1 \cdot e^{9x} dx =$$

$$1.) \text{ Set } u = x \quad v'(x) = e^{9x}$$

$$u' = 1 \quad v(x) = \int e^{9x} dx = \frac{1}{9} e^{9x}$$

$$= \frac{8x}{9} \cdot e^{9x} - \frac{8}{9} \int e^{9x} dx = \frac{8x}{9} \cdot e^{9x} - \frac{8}{9} \cdot \frac{1}{9} e^{9x} + C = \frac{8x}{9} e^{9x} - \frac{8}{81} e^{9x} + C$$

no (u)

Let us check if $\frac{d}{dx} \left(\frac{8x}{9} e^{9x} - \frac{8}{81} e^{9x} + C \right) \stackrel{?}{=} 8xe^{9x}$

$$= \left(\frac{8x}{9} e^{9x} \right)' - \left(\frac{8}{81} e^{9x} \right)' + C' = \left(\frac{8}{9} \cdot e^{9x} + \frac{8x}{9} \cdot 9e^{9x} \right) - \frac{8}{81} (e^{9x})'$$

$$= \left(\frac{8}{9} e^{9x} + 8x e^{9x} \right) - \frac{8}{81} \cdot 9e^{9x} + 0 = \frac{8}{9} e^{9x} + 8xe^{9x} - \frac{8}{9} e^{9x} = 8xe^{9x}$$

b.) $\int x \sin(3x) dx = -\frac{x}{3} \cos(3x) - \int (-\frac{1}{3}) \cos 3x dx =$

Formula: $\int u dx = uv - \int v du$

1. Set $u = x, \int dv = \sin(3x) dx = u + v(x)$

$du = 1 \cdot dx, v = -\frac{1}{3} \cos(3x) = uv + C$

$= -\frac{x}{3} \cos 3x + \frac{1}{3} \int \cos(3x) dx = -\frac{x}{3} \cos 3x + \frac{1}{3} \cdot \frac{1}{3} \sin(3x) + C$

$= -\frac{x}{3} \cos 3x + \frac{1}{9} \sin(3x) + C$

$\int (x^n)^m dx = \frac{x^{n+1}}{n+1} \cdot \frac{1}{m} = \frac{x^{n+1}}{(n+1)m} = \frac{x^{n+1}}{m(n+1)}$

Integration by parts

Integration by parts

Integration by parts

Integration by parts

Integration by parts

$x^b(x)^n v = v(x)^n \cdot x^b v - v(x)^n = v(x)^n \cdot x^b v$

$x^b(x)^n v = v(x)^n$

$(x)^n$

$= \left((x)^n v(x)^n \right)' = (x)^n v(x)^n + n(x)^{n-1} v(x)^n$

$= x^b (x)^n v(x)^n + n(x)^{n-1} v(x)^n$

$x^b (x)^n v(x)^n + x^b (x)^n v(x)^n =$

$= x^b (x)^n v(x)^n + x^b (x)^n v(x)^n = x^b v(x)^n$

$x^b (x)^n v(x)^n = x^b v(x)^n$

$x^b (x)^n v(x)^n = x^b v(x)^n$

Integration by parts

Integration by parts

$= x^b x^p \cdot \frac{1}{p} x^{p-1} = x^b x^p \cdot \frac{1}{p} x^{p-1} = x^b x^p \cdot \frac{1}{p} x^{p-1}$

$x^p = (x)^p, x = N + C$

$x^p \cdot \frac{1}{p} = x^p \cdot \frac{1}{p} = (x)^p \cdot \frac{1}{p} = N$

$\int x^p dx = \frac{x^{p+1}}{p+1} + C = \int x^p dx = \frac{x^{p+1}}{p+1} + C = \frac{x^{p+1}}{p+1} + C$