where I is an interval, then $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$, $g(x) = u$

Ex: Use the Substitution method to evaluate the definite integral.

1.) $\int_0^1 6x^5 (\boxed{1+x^6})^5\,dx = \int \cancel{6x^5}(u)^5 \frac{du}{\cancel{6x^5}} = \int_1^2 u^5\,du = \Rightarrow$

1.) Set $u = 1 + x^6$

2.) $\frac{du}{6x^5} = \frac{6x^5\,dx}{6x^5} = dx = \frac{du}{6x^5}$

3.) If $x = 0$, use $u = 1 + x^6$, $u = 1 + 0^6 = \boxed{1}$

If $x = 1$, $u = 1 + x^6$, $u = 1 + 1^6 = \boxed{2}$

$\Rightarrow = \frac{u^6}{6}\Big|_1^2 = \frac{2^6}{6} - \frac{1^6}{6} = \frac{64}{6} - \frac{1}{6} = \boxed{63/6}$

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This is not on this coming test.

Section 7.2: Intergration by Parts & Practicing Intergration

Intergration by Parts: Let $u(x)$ & $v(x)$ be differentiable functions. Then $\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$.

or $\int u\,dv = uv - \int v\,du$. $\Rightarrow dv = v'(x)\,dx$

$du = u'(x)\,dx$

$u(x)$

Proof: Consider $(u(x)v(x))' = \frac{d}{dx}(u(x)v(x)) =$

$u(x)\,v'(x) + u'(x)v(x)$. $\int (u(x)v(x))'\,dx =$

$= \int u'(x)\,v(x)\,dx + \int u(x)\,v'(x)\,dx$

$u(x)\,v(x) = \int u'(x)v(x)\,dx + \int u(x)\,v'(x)\,dx$

$- \int u'(x)v(x)\,dx \quad - \int u'(x)v(x)\,dx$

We have $\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$

Ex: Evaluate using intergration by parts or substitution. Check by differentiating.

a.) $\int 8xe^{9x}\,dx = 8\int x e^{9x}\,dx = 8x \cdot \frac{1}{9}e^{9x} - 8\int 1 \cdot e^{9x}\,dx =$

1.) Set $u = x \quad v'(x) = e^{9x}$

$u' = 1 \quad v(x) = \int e^{9x}\,dx = \frac{1}{9}e^{9x}$

$= \frac{8x}{9} \cdot e^{9x} - \frac{8}{9}\int e^{9x}\,dx = \frac{8x}{9} \cdot e^{9x} - \frac{8}{9} \cdot \frac{1}{9}e^{9x} + C = \boxed{\frac{8x}{9}e^{9x} - 8/81\, e^{9x} + C}$

Let us check if $\frac{d}{dx}\left(\frac{8x}{9}e^{9x} - \frac{8}{81}e^{9x} + C\right) \stackrel{?}{=} 8xe^{9x}$

$= (8x/9\, e^{9x})' - (\frac{8}{81}e^{9x})' + C' = \left(\frac{8}{9}\cdot e^{9x} + \frac{8x}{9}\cdot 9e^{9x}\right) - \frac{8}{81}(e^{9x})'$

$= \left(\cancel{\frac{8}{9}e^{9x}} + \frac{8x}{\cancel{9}}\cancel{9}e^{9x}\right) - \cancel{\frac{8}{81}\cdot 9e^{9x}} + 0 = \boxed{8xe^{9x}}$ $= 8/81 \cdot 9e^{9x}$

b.) $\int x\sin(3x)\,dx = \frac{-x}{3}\cos(3x) - \int(-1/3)\cos 3x\,dx =$

formula: $\int u\,dv = uv - \int v\,du$

1. Set $u = x$, $\int dv = \int \sin(3x)\,dx$

$du = 1\cdot dx$, $v = -1/3\cos(3x)$

$= \frac{-x}{3}\cos 3x + 1/3\int\cos(3x)\,dx = \frac{-x}{3}\cos 3x + 1/3\cdot 1/3\sin(3x) + C$

$= \boxed{\frac{-x}{3}\cos 3x + 1/9\sin(3x) + C}$