Ex: p. 322 # 21 If $\frac{dl}{dt}$ represents the growth rate of an organism at time t (measured in months), explain what $\int_2^7 \frac{dl}{dt}\,dt$ represents

Solution: $\int_2^7 \frac{dl}{dt}dt = l(7) - l(2)$, just means net change of organism from second to seventh month.

Lecture 19-9-14 Professor Shabanskaya Calculus 1760-002

Next Friday we have test.

Section 7.1: The Substitution Rule

1.) Substitution Rule for Indefinate Integrals:

- Let $u = g(x)$ be a differentiable function with the range I, where I is some interval & f be continuous on I. Then $\int f(g(x))\,g'(x)\,dx =$ $\int f(u)\,du$ {$u = g(x)$ (replace $g(x)$ with u "so $\int f(u)\,g'(x)\,dx$) & $du = g'(x)\,dx$}

Ex: Use substitution to evaluate the indefinate integrals:

a.) $\int 3x e^{x^2}\,dx = 3\int x e^{(x^2)}\,dx = 3\int x e^u \frac{du}{2x} \Rightarrow$

1.) Set $u = x^2$

2.) $\frac{du}{2x} = \frac{2x\,dx}{2x} = dx = \frac{du}{2x}$

$\Rightarrow = 3\int x e^u \frac{du}{2x} = 3 \cdot 1/2 \int e^u\,du = 3/2\, e^u + C \overset{u=x^2}{=} \boxed{3/2\, e^{x^2} + C}$

b.) $\int 2x\sqrt{1+2x^2}\,dx$ $u = 1+2x^2$ $\int \frac{2x}{u}\frac{(du)}{(dx)} = \int x \cdot \frac{1}{u}\,dx \Rightarrow$

1.) Set $u = 1+2x^2$

2. $\frac{du}{4x} = \frac{4x\,dx}{4x} = dx = \frac{du}{4x}$

$\Rightarrow = \int \frac{2x}{u}\frac{du}{4x} = 1/2 \int 1/u\,du = 1/2 \ln|u| + C \overset{u=1+2x^2}{=} 1/2 \ln|1+2x^2| + C =$ $\Leftarrow > 0$

$\Rightarrow = \boxed{1/2 \ln(1+2x^2) + C}$

2.) Substitution Rule for Definate Integrals:

If $g'(x)$ is continuous on the closed interval $[a,b]$ & f is continuous on the range of g say I,

where I is an interval, then $\int_a^b F(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} F(u)\,du$ ($g(x)=u$)

Ex: Use the Substitution method to evaluate the definate integral.

1.) $\int_0^1 6x^5\,(1+x^6)^5\,dx = \int 6x^5\,(u)^5\,\frac{du}{6x^5} = \int_{a1}^{2x} u^5\,du = \Rightarrow$

1.) Set $u = 1+x^6$

2.) $\frac{du}{6x^5} = \frac{6x^5\,dx}{6x^5} = dx = \frac{du}{6x^5}$

3.) If $x=0$, use $u=1+x^6$, $u=1+0^6=\boxed{1}$

If $x=1$, $u=1+x^6$, $u=1+1^6=\boxed{2}$

$\Rightarrow = \frac{u^6}{6}\Big|_1^2 = \frac{2^6}{6} - \frac{1^6}{6} = \frac{64}{6} - \frac{1}{6} = \boxed{\frac{63}{6}}$