

Ex: p. 322 # 21 If $\frac{dl}{dt}$ represents the growth rate of an organism at time t (measured in months), explain what $\int_2^7 \frac{dl}{dt} dt$ represents

Solution: $\int_2^7 \frac{dl}{dt} dt = l(7) - l(2)$, just means net change of organism from second to seventh month.

Lecture 19-9-14 Professor Shabanaskaya Calculus 1760-002

Next Friday we have test.

Section 7.1: The Substitution Rule

1.) Substitution Rule for Indefinite Integrals:

- Let $u = g(x)$ be a differentiable function with the range I , where I is some interval & f be continuous on I . Then $\int f(g(x))g'(x) dx = \int f(u) du$ ($u = g(x)$ replace $g(x)$ with u so $du = g'(x) dx$)

Ex: Use substitution to evaluate the indefinite integrals:

a.) $\int 3x e^{x^2} dx = 3 \int x e^{x^2} dx = 3 \int x e^u dx \Rightarrow$

1.) Set $u = x^2$

2.) $\frac{du}{2x} = 2x dx = dx = \frac{du}{2x}$

$\Rightarrow = 3 \int x e^u \frac{du}{2x} = 3 \cdot \frac{1}{2} \int e^u du = \frac{3}{2} e^u + C = \frac{3}{2} e^{x^2} + C$

b.) $\int \frac{2x}{1+2x^2} dx$; $u = 1+2x^2$; $\int \frac{2x}{u} \frac{du}{2x} = \int \frac{1}{u} du \Rightarrow$

1.) Set $u = 1+2x^2$

2. $\frac{du}{4x} = \frac{4x dx}{4x} = dx = \frac{du}{4x}$

$\Rightarrow = \int \frac{1}{u} \frac{du}{2} = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x^2| + C =$
 $\Rightarrow = \frac{1}{2} \ln(1+2x^2) + C$

2.) Substitution Rule for Definite Integrals:

If $g'(x)$ is continuous on the closed interval $[a, b]$ & f is continuous on the range of g say I ,

where I is an interval, then $\int_a^{g(b)} F(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} F(u) du$
 $g(x) = u$

Ex: Use the Substitution method to evaluate the definite integral.

$$1.) \int_0^1 6x^5 (1+x^6)^5 dx = \int_1^2 6x^5 (u)^5 \frac{du}{6x^5} = \int_1^2 u^5 du = \Rightarrow$$

$$1.) \text{ Set } u = 1 + x^6$$

$$2.) \frac{du}{6x^5} = \frac{6x^5 dx}{6x^5} = dx = \frac{du}{6x^5}$$

$$3.) \text{ If } x = 0, \text{ use } u = 1 + x^6, u = 1 + 0^6 = \boxed{1}$$

$$\text{If } x = 1, u = 1 + x^6, u = 1 + 1^6 = \boxed{2}$$

$$\Rightarrow = \frac{u^6}{6} \Big|_1^2 = \frac{2^6}{6} - \frac{1^6}{6} = \frac{64}{6} - \frac{1}{6} = \boxed{\frac{63}{6}}$$