Lecture: 15-9-14 Proffersor Shabanskaya Calculus: 1760-002

Blackboard has quiz solutions, MyMathLab has homework & quiz grades

Section 6.2 The Fundamental Theorem of Calculus

Let f be a continuous function on $[a,b]$. Then

Part I: Let $F(x) = \int_a^x f(x)\,dx$, $a \le x \le b$. Then $F(x)$ is continuous on $[a,b]$, & differentiable on (a,b) and $\frac{d}{dx}F(x) = f(x)$ ($F(x)$ is an antiderivative of $f(x)$)

Part II: $\int_a^b f(x)\,dx = \underbrace{F(x)}_{\text{an antiderivative}}\Big|_a^b = F(b) - F(a)$

Ex: Calculate the definate intergrals.

1.) $\int_3^6 \frac{8}{x}\,dx = 8\int_3^6 \frac{1}{x}\,dx = 8\ln|x|\Big|_3^6 = \rightarrow$

$\int 1/x\,dx = \ln|x| + C$ (dont need constant

$\rightarrow 8\ln 6 - 8\ln 3 = 8(\ln 6 - \ln 3) = 8\ln\frac{6}{3} = \boxed{8\ln 2}$

2.) $\int_{-1}^{3}(4x^2 - 5x + 4)\,dx = \int_{-1}^{3} 4x^2\,dx - \int_{-1}^{3} 5x\,dx + \int_{-1}^{3} 4\,dx =$

$= 4\int_{-1}^{3} x^2\,dx - 5\int_{-1}^{3} x\,dx + 4\int_{-1}^{3} 1\,dx = 4\frac{x^3}{3}\Big|_{-1}^{3} - 5\frac{x^2}{2}\Big|_{-1}^{3} + 4x\Big|_{-1}^{3} =$

$= \left(\frac{4(3)^3}{3} - \frac{5(3)^2}{2} + 4(3)\right) - \left(\frac{4(-1)^3}{3} - \frac{5(-1)^2}{2} + 4(-1)\right) \stackrel{?}{=} \frac{108}{3}$

3.) $\int_4^{16} \frac{5t+2}{8\sqrt{t}}\,dt = \frac{5t}{8\sqrt{t}} + \frac{2}{8\sqrt{t}} = \frac{5}{8}\frac{t}{t^{1/2}} + \frac{1}{4}\frac{1}{t^{1/2}} =$

$= 5/8\, t^{-1/2} \cdot t + 1/4\, t^{-1/2} = \boxed{5/8\, t^{1/2} + 1/4\, t^{-1/2}} =$

$= \int_4^{16} 5/8\, t^{1/2}\,dt + \int_4^{16} 1/4\, t^{-1/2}\,dt = 5/8\int_4^{16} t^{1/2}\,dt + 1/4\int_4^{16} t^{-1/2}\,dt =$

$= 5/8\,\frac{t^{1/2+1}}{1/2+1}\Big|_4^{16} + 1/4\,\frac{t^{-1/2+1}}{-1/2+1}\Big|_4^{16} = 5/8 \cdot 2/3 \cdot t^{3/2}\Big|_4^{16} + 1/4 \cdot 2 \cdot t^{1/2}\Big|_4^{16}$

$= 5/12\, t^{3/2}\Big|_4^{16} + 1/2\, t^{1/2}\Big|_4^{16} = \frac{5}{12}\cdot 16^{3/2} + 16^{1/2} \cdot 1/2) - (\frac{5}{12}\cdot 4^{3/2} + 1/2\, 4^{1/2})$

$\frac{3}{2}$?

Leibniz's Rule: Let $g(x)$ & $h(x)$ be differentiable functions & $f(u)$ be a continuous function for $g(x) \le u \le h(x)$. Then $\frac{d}{dx}\int_{g(x)}^{h(x)} f(u)\,du = f(h(x))\,h'(x) - f(g(x))\cdot g'(x)$

II $\int_{g(x)}^{h(x)} f(u)\,du = \int_{g(x)}^{0} f(u)\,du + \int_0^{h(x)} f(u)\,du = \int_0^{h(x)} f(u)\,du = F(h(x)) - F(g(x))$

$-\int_0^{g(x)} f(u)\,du$