Then set $F(g(x)) = \int_0^{g(x)} f(u)\,du$ and $F(h(x)) =$
~~let~~ $\int_0^{h(x)} f(u)\,du$

Find $\frac{d}{dx}\left(F(h(x)) - F(g(x))\right) = F' = f$

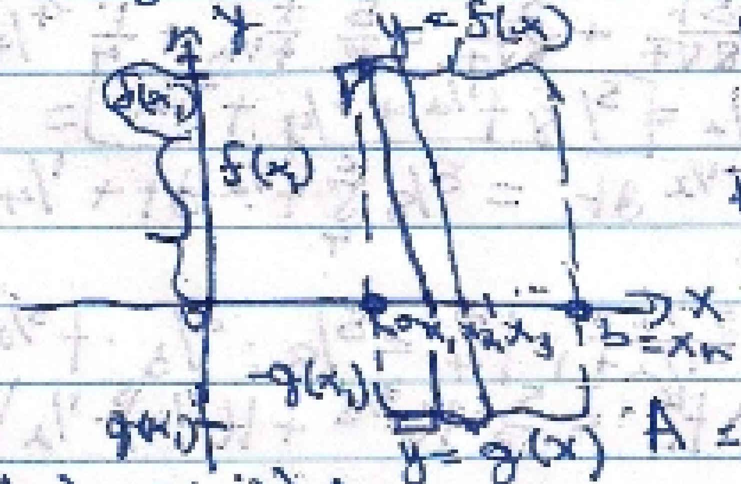
Ex: p305 Use Leibniz rule to find $\frac{d}{dx}$
15) $y = \int_0^{3x} (1+t^3)\,dt$
Solution: 1.) $f(x) = 1+t^3$ is continuous on $[0,\infty)$
2.) $\frac{dy}{dx} = (1+(3x)^3)\cdot\frac{d}{dx}(3x) = 3(1+9x^2)$
Homework: P. 305 # 10, 11, 18, 19, 33, 37

Lecture: 17-9-14 Proffessor Shabanskaya Calculus 1760-002
Quiz 3 on friday, covers homeworks 9 & 10, 2 problems, 10 minutes

Section 6.3: Applications of Definate Integrals
Let f & g be continuous functions on $[a,b]$ such that $f(x) \geq g(x)$. Then the area between $f(x)$ and $g(x)$ on $[a,b]$ denoted

$$A = \int_a^b (f(x) - g(x))\,dx \geq 0$$

$f(x) \geq g(x)$ can be written as $f(x) - g(x) \geq 0$



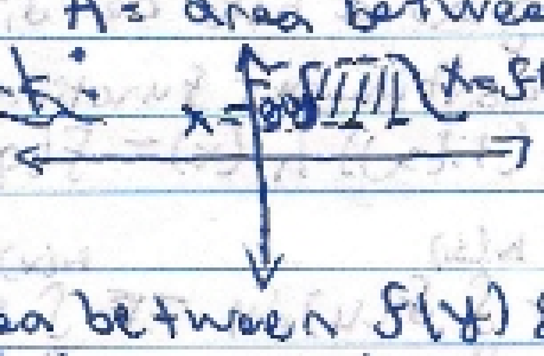
Divide $[a,b]$ into n equal parts with the length of each subinterval $\Delta x = \frac{b-a}{n}$

$A \approx (f(x_i) - g(x_i))\Delta x$

$$A = \sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x \leftarrow \text{sum of areas of rectangles}$$

$n \to \infty$ $A =$ area between $f(x)$ & $g(x)$ on $[a,b]$

Remark:



If we have two curves $f(y)$ & $g(y)$ such that $f(y) \geq g(y)$, $c \leq y \leq d$
The area between $f(y)$ & $g(y)$ is denoted by
$A = \int_c^d (f(y) - g(y))\cdot dy$

Examples: Find the area of the region bounded by the graph of the given equations: $y=4x+5$ & $y=x^2$

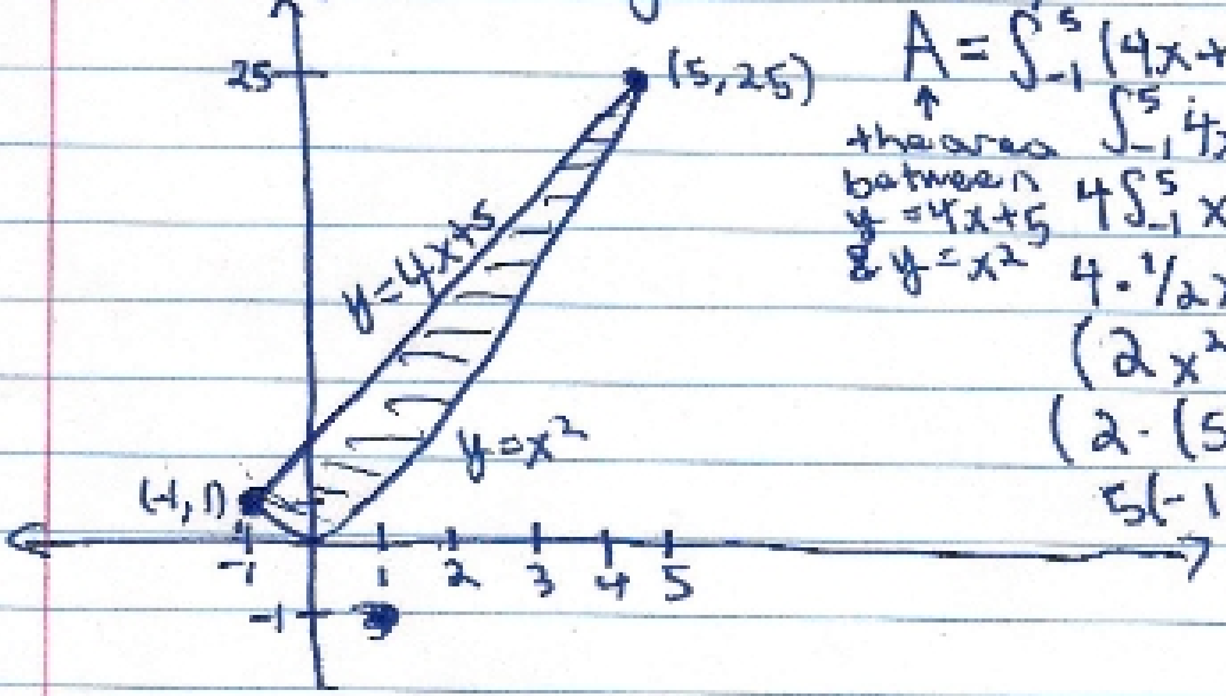
Solutions: 1.) Find the points of the intersection of two graphs $y=4x+5$ & $x^2=y$ (set equal to eachother)

a. $4x+5=x^2 = x^2-4x-5=0$ & $x_{1,2}=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm 6}{2}= x_1=5$, $x_2=-1$

To find y coordinate substitute x into equation $4x+5=y$ or $x^2=y$

b. $x_1=5$, $5^2=25$ $y=25$, $(5,25)$

c $x_2=-1$, $-1^2=1$ $y=1$ $(-1,1)$



$A=\int_{-1}^{5}(4x+5-x^2)\,dx=$

the area between $y=4x+5$ & $y=x^2$

$\int_{-1}^{5}4x\,dx+\int_{-1}^{5}5\,dx-\int_{-1}^{5}x^2\,dx=$

$4\int_{-1}^{5}x\,dx+5\int_{-1}^{5}1\,dx-\int_{-1}^{5}x^2\,dx=$

$4\cdot\frac{1}{2}x^2+5\cdot x-\frac{1}{3}x^3=$

$(2x^2+5x-\frac{1}{3}x^3)\Big|_{-1}^{5}=$

$(2\cdot(5)^2+5\cdot5-\frac{1}{3}(5^3))-(2\cdot(-1)^2+5(-1)-\frac{1}{3}(-1)^3)=36$

$\frac{220}{3}-(\frac{8}{3})$

Net/Cumulative Change

Recall (FTC Part II) If f is continuous on $[a,b]$, then $\int_a^b f(x)\,dx=F(x)\Big|_a^b=F(b)-F(a)$ where $\underbrace{\frac{d}{dx}F(x)}_{F'(x)}=f(x)$

Plug in $F'(x)$ instead of $f(x)$. We have $\int F'(x)\,dx=F(b)-F(a)$, $F(b)-F(a)$ is the net/cumulative change of $F(x)$ on $[a,b]$, and $F'(x)$ is the rate of change of $F(x)$.

The definite intergral of the rate of change of $F(x)$ on $[a,b]$ = Net change (cumulative change) of $F(x)$ on $[a,b]$

Ex: P. 322 # 21 If $\frac{dl}{dt}$ represents the growth rate of an organism at time t (measured in months), explain what $\int_2^7 \frac{dl}{dt}\,dt$ represents

<u>Solution</u>: $\int_2^7 \frac{dl}{dt}dt = l(7) - l(2)$; just means net change of organism from second to seventh month.

"$l'(x)$"