

Final Exam
Math 274
December 19, 2000

Name _____

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam. The use of a graphing calculator is permitted.

§1 Calculation

1. Evaluate

a. $\int_0^1 \tan^{-1} x \, dx$

b. $\int_{\sqrt{2}}^2 \frac{dt}{t^3 \sqrt{t^2 - 1}}$

2. Find the length of the arc of the semicubical parabola $y^2 = x^3$ from (1,1) to (4,8).

3. Find the volume of the solid obtained by revolving the region bounded by $y = x - x^2$ and $y = 0$ rotated about the line $x = 2$.

4. The arc of the circle $y = \sqrt{4 - x^2}$ between $x = -1$ and $x = 1$ is rotated about the x -axis. Find the area of the resulting surface.

5. Use Simpson's rule with 6 intervals to approximate $\int_0^1 e^{-x^2} \, dx$.

6. Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

a. $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{1+n^4}$

b. $\sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 1}}{3n^3 + 4n^2 + 1}$

7. Estimate $\sum_{n=1}^{\infty} \frac{1}{n^4}$ accurately to within 0.01.

8. Evaluate the integral $\int \sqrt{x^3 + 1} \, dx$ by using power series. Give the radius of convergence of your result.

§2 Comprehension

9. What is the direction field for a differential equation? Sketch the direction field for $y' = y(2 - y)$, and draw in some representative solutions.

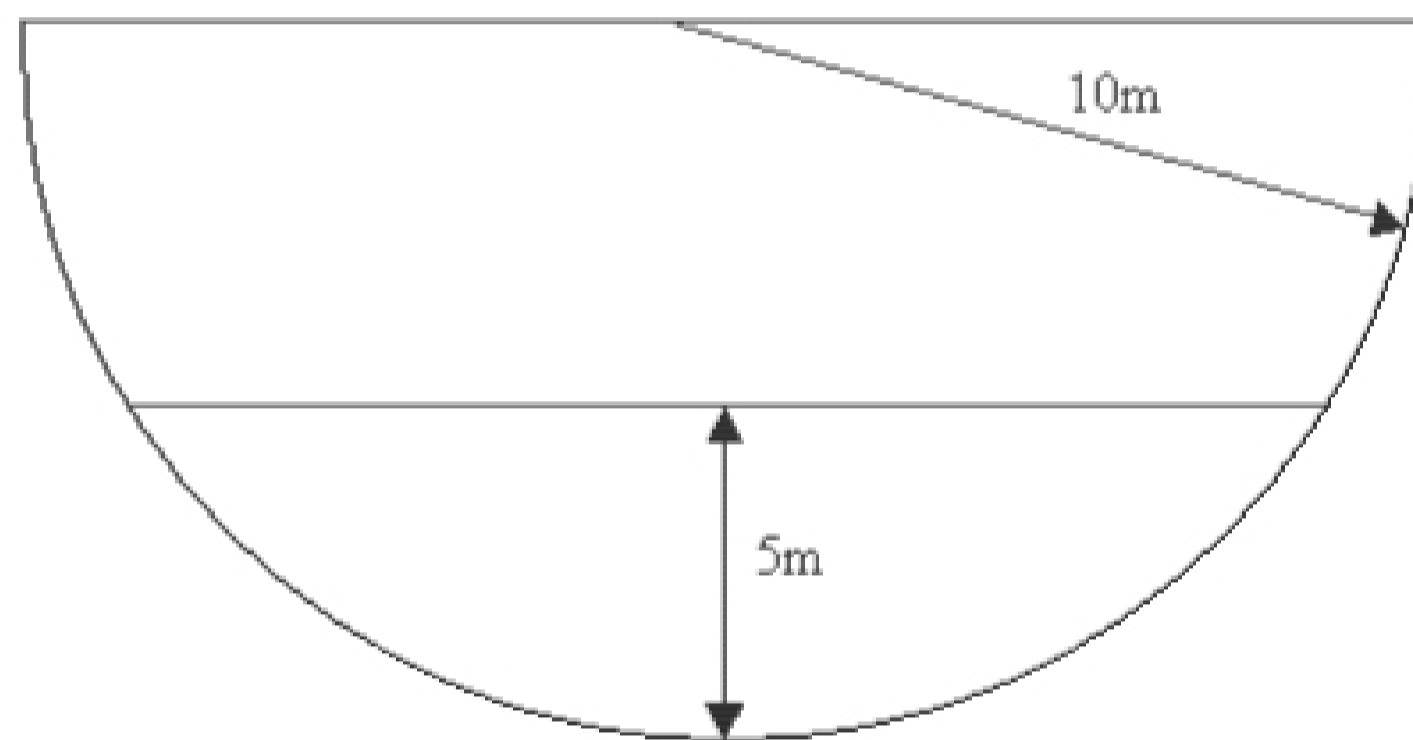
10. What is a series? What does it mean for an infinite series to converge.

11. List and briefly describe five different tests for the convergence of an infinite series.

12. State Taylor's theorem precisely

§3 Application

13. A 1600 lb. elevator is suspended by a 200 ft. cable that weighs 10 lb./ft. How much work is needed to raise the elevator from the basement to the third floor, a distance of 30 feet?
14. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 30 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after t minutes?
15. The end of a tank is vertical in the shape of a semicircle with radius 10m. If the bottom 5m of the tank is filled with water, what is the force of the water on the side of the tank?



16. The electric potential V at a distance R along the axis perpendicular to the center of a charged disc with radius a and charge density σ is given by

$$V = 2\pi\sigma(\sqrt{R^2 + a^2} - R).$$

If R is much larger than a , show that

$$V \approx \frac{\pi a^2 \sigma}{R}.$$

Estimate the error.