

1. Approximate the area under the graph $y = \frac{x-1}{x+1}$ over $[0, 6]$ using 3 intervals ($n = 3$) and midpoints.

(a) $\frac{7}{3}$

(b) $\frac{11}{12}$

(c) $\frac{5}{6}$

(d) $\frac{7}{6}$

(e) $\frac{5}{12}$

(f) $\frac{5}{3}$

The intervals are $[0, 2], [2, 4], [4, 6]$, so the mid points are 1, 3, 5, $\Delta x = \frac{6-0}{3} = 2$.

$$\text{area} \approx \frac{1-1}{1+1} \cdot 2 + \frac{3-1}{3+1} \cdot 2 + \frac{5-1}{5+1} \cdot 2 = 0 + 1 + \frac{8}{6} = \frac{14}{6} = \frac{7}{3}$$

2. Which of the following is the formula for the Riemann sum $f(x) = x^3$ over $[1, 2]$ with n -subintervals and right end points?

(a) $\sum_{i=0}^n \frac{1}{n} \left(1 + \frac{1+i}{n}\right)^3$

(b) $\sum_{i=0}^n \frac{2}{n} \left(1 + \frac{i}{n}\right)^3$

(c) $\sum_{i=1}^n \frac{1}{n} \left(1 + \frac{i}{n}\right)^3$

(d) $\sum_{i=1}^n \frac{2}{n} \left(1 + \frac{1+i}{n}\right)^3$

(e) $\sum_{i=0}^{n-1} \frac{1}{n} \left(1 + \frac{i}{n}\right)^3$

(f) $\sum_{i=0}^{n-1} \frac{2}{n} \left(1 + \frac{1+i}{n}\right)^3$

3. Evaluate

$$\int_1^4 \frac{3x-1}{\sqrt{x}} dx.$$

(a) $\frac{11}{3}$

(b) 2

(c) -2

(d) $\frac{8}{3}$

(e) $\frac{3}{2}$

(f) 12

$$\int_1^4 \frac{3x-1}{\sqrt{x}} dx = \int_1^4 3x^{+\frac{1}{2}} - x^{-\frac{1}{2}} dx = 2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} \Big|_1^4 = (2 \cdot 8 - 2 \cdot 2)$$

$$-(2 \cdot 2) = 16 - 4 = 12$$