

Calculus II Notes (MATH 222)

Lauren J. Kopac

Spring 2013

Contents

Calculus II Contents	4
1 Integration and Differentiation	4
1.1 Integration by Substitution	4
1.1.1 Technique	4
1.1.2 Examples: Evaluate	5
1.2 Exponential Functions	6
1.2.1 Exponential Rules	6
1.2.2 Exponential Functions	7
1.2.3 The Natural Base	7
1.2.4 Compound Interest	7
1.2.5 Continuous Compounding	8
1.2.6 Differentiation of e^x	8
1.2.7 The General Form	9
1.2.8 Problems: Differentiate	9
1.2.9 Integration of e^x	10
1.2.10 Problems: Integrate	10
1.3 Logarithms	11
1.3.1 Inverse Functions	11
1.3.2 Question: What about x^2 and \sqrt{x} ?	11
1.3.3 Derivative of an Inverse Function	12
1.3.4 Example: Find the derivative of \sqrt{x}	12
1.3.5 Logarithms	12
1.3.6 Differentiation of $\ln x$	13
1.3.7 General Form	13
1.3.8 Problems: Differentiate	14
1.3.9 Integration	14
1.3.10 Problems: Integration	14
1.4 Other Bases and Inverse Trig Functions	15

1.4.1	Other Bases	15
1.4.2	Summary of Other Bases	15
1.4.3	Problems: Evaluate	16
1.4.4	Derivatives of Inverse Trig Functions	17
1.4.5	Integrals of Inverse Trig.	18
1.4.6	Problems: Differentiate	18
1.4.7	Problems: Integrals	18
1.5	Logarithmic Differentiation and L'Hôpital's Rule	19
1.5.1	Technique: Logarithmic Differentiation	19
1.5.2	Problems: Find $\frac{dy}{dx}$	19
1.5.3	L'Hôpital's Rule	20
1.5.4	Problems: Evaluates	20
1.6	Integration by Parts	21
1.6.1	Problems: Integrate	21
1.6.2	Technique	21
1.6.3	Technique: Integration by Parts	22
1.6.4	Problems: Integrate	22
1.6.5	Problems: Evaluate	23
1.6.6	Definite Integral Rule	23
1.7	Trigonometric Techniques	23
1.7.1	Basic Trig Integration Rules	23
1.7.2	Technique: Integrating $\int \sin^m x \cos^n x dx$	24
1.7.3	Problems: Integrate	24
1.7.4	Technique: Integrating $\int \tan^m x \sec^n x dx$	25
1.7.5	Problems: Integrate	25
1.7.6	Special Right Triangles	25
1.7.7	Problems: Integrate	26
1.8	Integration by Partial Fractions	26
1.8.1	Rule	26
1.8.2	Technique: Integration by Partial Fraction Decomposition	26
1.8.3	Problems: Integrate	27
1.9	Improper Integrals	27
1.9.1	Problem: Evaluate	28
1.9.2	Definition	28
1.9.3	Problems: Evaluate	29
1.10	Numerical Integration	29
1.10.1	Trapezoidal Method	29
1.10.2	Simpson's Rule	30
2	Basic Differential Equations	30
2.1	Introduction to Differential Equations	30
2.1.1	Definition	30
2.1.2	Technique: Solving Separable Differential Equations	31

2.1.3	Problems: Solve	31
2.2	Euler's Method	32
2.2.1	Growth and Decay	32
2.2.2	Question: Can we solve for $y(t)$?	32
2.2.3	Problems	33
2.2.4	Radio Active Decay	33
2.2.5	Half-Life	34
2.2.6	Problems	34
2.2.7	Euler's Method	35
2.2.8	Technique: Euler's Method	35
3	Sequences and Series	36
3.1	Sequences	36
3.1.1	Examples	36
3.2	Series	37
3.2.1	Partial Sum	37
3.2.2	Infinite Series	37
3.3	Geometric Series and the Integral Test	38
3.3.1	Definition	38
3.3.2	Examples	38
3.3.3	Geometric Series Rule	38
3.3.4	Problems: Converge or Diverge?	38
3.3.5	Integral Test	39
3.4	Comparison Tests and the Ratio Test	39
3.4.1	The Comparison Test	39
3.4.2	Problem: Determine Convergence or Divergence	40
3.4.3	The Limit Comparison Test	40
3.5	The Ratio Test	40
3.5.1	Useful Limits	40
3.5.2	Technique	41
3.6	The Power Series	42
3.6.1	Definition	42
3.6.2	Convergence/Divergence of Power Series	42
3.6.3	Summary	43
3.7	Root Alternating Series Test	43
3.7.1	The Root Test	43
3.7.2	Alternating Series Test	44
3.8	Taylor Series	45
3.8.1	Common Taylor Series	45
3.9	Distance Along a Curve	45
3.9.1	Straight-Line Distance	45
3.9.2	Cartesian Form	46
3.9.3	Parametric Form	46