1. What is the average value of the following function over $[0, \pi]$?

$$f(x) = \sin x - \sin 2x$$

(a) 0

(b) 2

(c) 4

(d) $\frac{1}{\pi}$

(e) $\frac{2}{\pi}$

(f) $\frac{4}{\pi}$

$$\text{average} = \frac{1}{\pi} \int_0^{\pi} (\sin x - \sin 2x)\, dx = \frac{1}{\pi} \left(-\cos x + \frac{\cos 2x}{2} \Big|_0^{\pi} \right)$$

$$= \frac{1}{\pi} \left(-\cos \pi + \frac{\cos 2\pi}{2} - \left(-\cos 0 + \frac{\cos 0}{2} \right) \right)$$

$$= \frac{1}{\pi} \left(1 + \frac{1}{2} + 1 - \frac{1}{2} \right) = \frac{2}{\pi}$$

2. Find c in $[0, 2]$ such that $f(c)$ is the average value of the function $f(x) = x^2$ over $[0, 2]$.

(a) $\frac{2}{3}$

(b) $\frac{4}{3}$

(c) 1

(d) $\sqrt{\frac{4}{3}}$

(e) $\sqrt{\frac{8}{3}}$

(f) $\sqrt{\frac{12}{3}}$

$$\text{average} = \frac{1}{2}\int_0^2 x^2\,dx = \frac{1}{2}\left(\frac{x^3}{3}\Big|_0^2\right) = \frac{1}{2}\left(\frac{8}{3}\right) = \frac{4}{3}$$

$$f(c) = \frac{4}{3} \qquad c^2 = \frac{4}{3} \qquad c = \sqrt{\frac{4}{3}}$$

3. What is the volume of the solid generated by rotating the region between the curve $y = \sqrt{x-1}$, $x = 5$, and $y = 0$ about the x-axis?

(a) $\frac{2\pi}{5}$

(b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$

(d) 2π

(e) 4π

(f) 8π

$$\text{Volume} = \int_1^5 \pi \left(\sqrt{x-1}\right)^2 dx = \int_1^5 \pi (x-1)\, dx = \frac{\pi x^2}{2} - \pi x \Big|_1^5$$

$$= \frac{25\pi}{2} - 5\pi - \frac{\pi}{2} + \pi = 8\pi$$