

MATHEMATICS 2210-3
Calculus III, Multivariable Calculus
Spring semester 2010

text: *Calculus, 9th edition* by Varberg, Purcell, Rigdon

when: MWF 10:45-11:35

where: AEB 320

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office hours (may change): M, W, F 2:10-3:00
and by appointment.

problem session: 10:45-11:45, SW 134

2210-3 home page: www.math.utah.edu/~korevaar/2210spring10

prerequisites: Grades of at least “C” in Math 1210 and 1220, or equivalent. Practically speaking, if you got a “C” grade in 1220 you are likely to struggle in this class.

course outline: This is the final course in the three-semester Calculus sequence, Mathematics 1210-1220-2210, and covers chapters 11-14 of the text. We assume you’ve already learned the material in chapters 1-10. As you probably realize, Calculus is part of the mathematical foundation with which science can model the world. Isaac Newton (1642-1727) was one of its co-discoverers, and his aim was to understand the physics he saw in the natural world, such as planetary motion. A beautiful quote of Galileo, from 1623, anticipates the mathematics which has followed:

Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it.

In single-variable Calculus the domain and range (input and output) of the functions you consider are both subsets of the real numbers. In real-world problems it is often more natural to consider multivariable possibilities, for the domain, the range, or both. Thus, if you want to model planetary (or other) motion, the input can be the real variable t , for time, but the output position function should either describe points in space (i.e. 3-dimensional real space, \mathbb{R}^3), or in a plane \mathbb{R}^2 containing the sun and the planet. (You’ve already discussed some parametric curves in the plane, see e.g. section 10.4 in the text.) If you want to study the temperature in Utah, the input could be described by (x, y, t) , where (x, y) are used to describe where you are in Utah, and t is time, and the output temperature $T(x, y, t)$ is a real number. If you want to describe electrical or magnetic fields, or complete weather systems, or any complicated system in science, engineering, business, medicine or industry, then the inputs and outputs are usually both multivariable. Our goal in Math 2210 is to adapt and extend the ideas of the derivative, the integral, and the Fundamental Theorem of Calculus to multivariable settings, and to study important applications which result.

It is a good idea to understand the geometry and algebra of the plane \mathbb{R}^2 , 3-space \mathbb{R}^3 , and n -space \mathbb{R}^n , in order to understand functions between them, and so we do this at the start of the course, in sections 11.1-11.4. A key algebraic and geometric concept in these sections is the notion of vectors. The rest of chapter 11 develops Calculus for functions with real-number input and multivariable (\mathbb{R}^n) output, such as those which describe particle motion. This is an important context to consider, and one you will return to in physics, as well as in Math 2250 or 2280, when you consider systems of ordinary differential equations. The primary calculus objects we consider will be the tangent (velocity) and acceleration vectors associated to particle motion, as well as something called the “curvature” which

measures how much the image curve (traced out by the particle) is bending. Time permitting, we will illustrate the value of our work by giving a version of Newton's famous deduction that *only* the inverse square law of gravitational acceleration is consistent with Kepler's observed laws of planetary motion - one of the great breakthroughs in the history of science.

Chapter 12, *Derivatives for Functions of Two or More Variables*, is a study of differential calculus for real-valued functions when the domain is multivariable and the range is one dimensional, the reverse of the situation in chapter 11. The notion of differentiability for such functions has a new twist based on linear approximation. We will meet new versions of the chain rule, critical points for optimization problems, and the second derivative test. Many of you will study functions with multivariable domains in courses about partial differential equations, for example in Math 3150, or in physics courses about electricity and magnetism.

Chapter 13, *Multiple Integrals*, is a study of definite integration for real valued functions of several variables, i.e. when the domain is a subset of the plane or 3-space rather than just an interval in the real numbers. It will turn out that we can reduce these integral problems to iterated 1-variable integrals, at which you are already experts. Understanding how to change variables in multiple integrals will be interesting. Applications of double and triple integrals include the computation of areas and surface areas, volumes, masses and moments.

Chapter 14, *Vector Calculus*, is analogous to the Fundamental Theorem of Calculus and its applications in 1210-1220. There are various versions of these generalizations, and they all relate integrals of certain (partial) derivatives of functions over domains in \mathbb{R}^n , to boundary integrals. Included in this zoo of results are Green's, Gauss', and Stoke's Theorems, which are the foundation of classical physics topics such as electro-magnetism and fluid mechanics.

grading: There will be two midterms, a comprehensive final examination, and homework. Each midterm will count for 20% of your grade, homework will count for 30%, and the final exam will make up the remaining 30%. The midterms are on February 17 and April 7. The final exam is at the University scheduled time, 10:30-12:30 a.m. on Tuesday May 4. **Note:** You must get a grade of at least "C" on the final exam in order to get a grade of "C" or higher in the course.

Homework will be assigned daily and collected weekly, on Wednesdays. Our grader will grade a subset of the problems you hand in. You are strongly encouraged to collaborate as necessary on homework problems, and to use whatever technology you find helpful. You must each complete and hand in your own problem set, however, and you will not learn the material if you just copy someone else's solutions. I use homework problems to let you drill basic skills, but also to explore more in-depth problems and applications. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned, so make sure that you really understand each problem you hand in.

Problem sessions: I will run optional problem sessions on Tuesdays from 10:45-11:45 in SW 134, and you are all invited. I know from previous experience that some students flourish without attending these sessions, but that others find them quite helpful; if you think you fall in the latter group you may wish to arrange your schedule so that you can attend the problem sessions. In addition to this special problem session, the Math Department Tutoring Center is located in Rushing Student Center between LCB and JWB. It is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m. to 6 p.m. on Friday (starting the second week of the semester). Some, but not all of the math tutors welcome questions from Math 2210 students. For more information consult the web address www.math.utah.edu/ugrad/tutoring.html .

It is the Math Department policy, and mine as well, to grant any withdrawal request until the University deadline of Friday March 5. Other important dates are Monday January 18, the last day to add this class, and Wednesday January 20, the last day to drop it. The American with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed,
daily subject matter approximated

M	11 Jan	11.1	3-space (\mathbb{R}^3), distance
W	13 Jan	11.2	vectors in \mathbb{R}^2 and \mathbb{R}^3
F	15 Jan	11.3	dot product algebra and geometry
M	18 Jan	none	Martin Luther King Day
W	20 Jan	11.3-11.4	and cross product algebra and geometry
F	22 Jan	11.4	continued
M	25 Jan	11.5	parametric curves and calculus
W	27 Jan	11.5	velocity and acceleration
F	29 Jan	11.6	lines and tangent lines
M	1 Feb	11.7	geometry of curves and relation to physics
W	3 Feb	11.7	curvature and the roller coaster equation for acceleration
F	5 Feb	11.8	cylinders and quadric surfaces
M	8 Feb	11.9	cylindrical and spherical coordinates
W	10 Feb	12.1	functions of several variables
F	12 Feb	12.2	partial derivatives
M	15 Feb	none	Presidents' Day
W	17 Feb	exam 1	11.1-12.2
F	19 Feb	12.3	limits and continuity
M	22 Feb	12.4	tangent planes, linear approximation, differentiability
W	24 Feb	12.5	directional derivatives and gradients
F	26 Feb	12.6	chain rule
M	1 Mar	12.7	differentiability and differential approximation (again!)
W	3 Mar	12.8	max-min problems
F	5 Mar	12.9	Lagrange multipliers for constrained max-min
M	8 Mar	13.1	double integrals over rectangles
W	10 Mar	13.2	iterated integrals
F	12 Mar	13.3	double integrals over non-rectangular domains
M	15 Mar	13.4	double integrals in polar coordinates
W	17 Mar	13.5	applications of double integrals
F	19 Mar	13.6	surface area