Test Total

Name _____

Final Exam Calculus III 3450:223:005 Dr. Norfolk Monday, December 11th, 2006
Show all of your work. Give exact, reasonably-simplified answers.

1. Evaluate the *line integral* $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the *vector field*

   $\mathbf{F}(x, y, z) = \sin(\pi x)\mathbf{i} + \cos(\pi y)\mathbf{j} + xz\mathbf{k}$ and C is the curve $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k},\ 0 \leq t \leq 1$

15 points

2. Show that $\mathbf{F}(x, y, z) = (2xy - \cos x)\mathbf{i} + (1 + x^2 + e^y)\mathbf{j}$ is a *conservative* field, by finding an associated *potential function*.

15 points

Page 1 Total (30 points)

3. Evaluate $\int_C \nabla f \cdot d\mathbf{r}$, where $f(x, y, z) = \tan\left(e^{xy+z}\right)$ and C is the curve $\mathbf{r}(t) = t^4\mathbf{i} - \cos t\mathbf{j} + \frac{1}{\pi}\sin^{-1} t\mathbf{k},\ 0 \leq t \leq \frac{1}{2}$.

15 points

4. Evaluate $\int_C -y^3dx + x^3dy$ where C is the boundary of the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$, traversed counterclockwise.

15 points

Page 2 Total (30 points)

5. Evaluate *curl* $\mathbf{F} = \nabla \times \mathbf{F}$ and *div* $\mathbf{F} = \nabla \cdot \mathbf{F}$, where $\mathbf{F}(x, y, z)$ is the vector field $\mathbf{F}(x, y, z) = (x^2 - 2y)\mathbf{i} + (x + 2z)^2\mathbf{j} + (x - y^2)\mathbf{k}$.

15 points

6. Evaluate the *flux* of the vector field $\mathbf{F}(x, y, z) = (x^2 + y^2)\mathbf{i} + xy\mathbf{j} - yz\mathbf{k}$ through the surface S of the region in the first octant bounded by $x^2 + y^2 = 4$ and $x + 2y + 3z = 6$.

15 points

Page 3 Total (30 points)