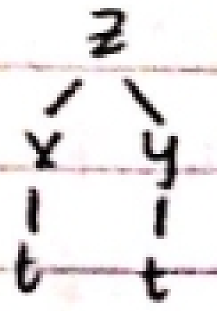
Calc Exam #2 Study Guide

- implicit differentiation

·yields $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ for whatever is in denominator, all other variables = fixed

w/ $\frac{\partial z}{\partial x}$, y is fixed and derivative = 0

Ex) $z = x+y \quad x=e^{t} \quad y=e^{-t}$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

- $D_{\vec{u}}f(x,y)$ = rate of change of f in given direction

$D_{\vec{u}}f = \nabla f \cdot \vec{u}$

$= \langle \frac{df}{dx}, \frac{df}{dy} \rangle \cdot \vec{u}$

·max rate of change is $|\nabla F|$

·sign of $D_{\vec{u}}f$ tells if incr/decr

- Finding the tangent line:

Ex) Tan line to int btw $x^2+y^2=2$, $z=3-x^2-y^2$ @ point (1, 1, 1)

$F(x,y,z) = x^2+y^2-2=0 \quad \nabla F = \langle 2x, 2y, 0\rangle \therefore \nabla F(1,1,1) = \langle 2,2,0\rangle$

$G(x,y,z) = z+x^2+y^2-3=0 \quad \nabla G = \langle 2x, 2y, 1\rangle \therefore \nabla G(1,1,1) = \langle 2,2,1\rangle$

·tan line is through (1,1,1) and orthogonal to ∇F and ∇G

·a vector along the line will be orthogonal to ∇F/∇G:

$$\begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 2 & 2 & 1 \end{vmatrix} = (2-0)i-(2-0)j+(4-4)k$$

$$= 2i-2j = \langle 2,-2,0\rangle$$

$$\therefore \boxed{\vec{r}(t) = \langle 1,1,1\rangle + t\langle 2,-2,0\rangle}$$

- second derivative tests:

·(a,b) is a critical point of f(x,y)

$D(a,b) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ if >0, 1) local max if $f_{xx}<0$ 2) local min if $f_{xx}>0$ if <0, saddle point bc no max/min if =0, no conclusion

① find critical points by doing $f_x/f_y=0$

② compute D

③ plug in crit. points to determine max/min

$\swarrow$ domain

- max/min on bounded functions - crit pts inside D / on boundary
- Lagrange multipliers (max/min given constraint)

$$\begin{cases} f_x = \lambda g_x + \mu h_x \\ f_y = \lambda g_y + \mu h_y \\ f_z = \lambda g_z + \mu h_z \\ g(x,y,z) = K \\ h(x,y,z) = K \end{cases} \left.\vphantom{\begin{matrix} a \\ b \end{matrix}}\right\} \text{constraints}$$

- Double integrals
  - for rectangles, $[x_1, x_2] \times [y_1, y_2]$ gives ranges of x/y
- Finding closest points:
  minimize dist formula: $d^2 = (\sqrt{x^2+y^2+z^2})^2$