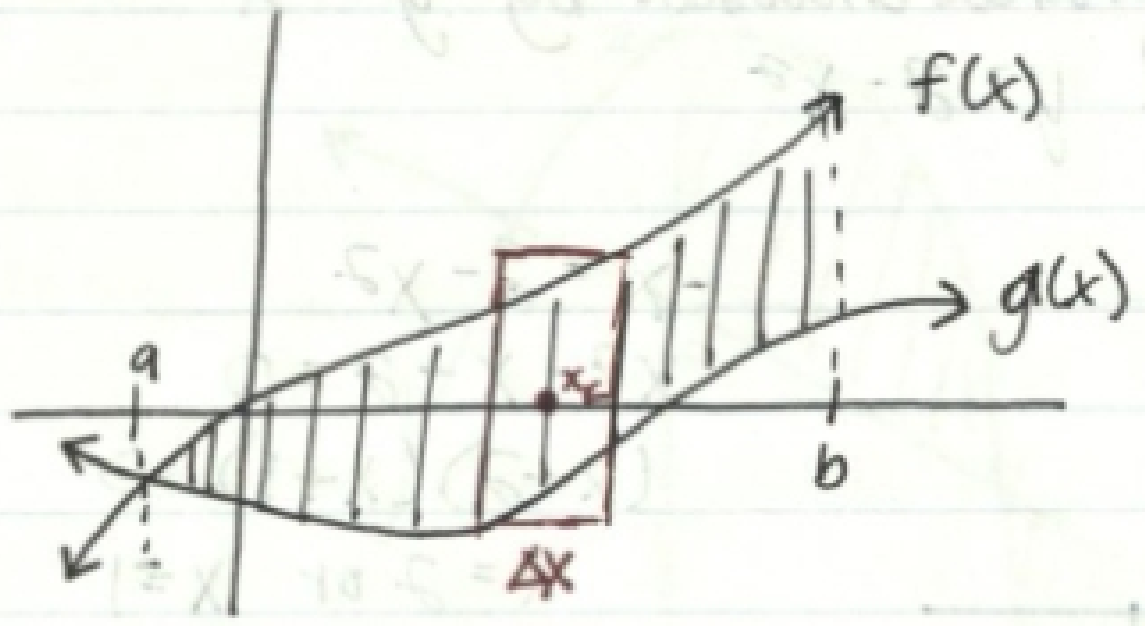
$\boxed{\text{Section 5.4}}$ Calculating Areas Between Two Curves over an Interval [a, b]



If we subdivide the interval [a,b] so that n rectangles are inscribed between the curves, each with a constant width of Δx then:

1. the area of a typical rectangle is:

$$\text{Area} = [\underbrace{f(x_k) - g(x_k)}_{\text{height}}] \underbrace{\Delta x}_{\text{width}}, \quad \Delta x = \frac{b-a}{n}$$

2. the estimated area between the curves over [a, b] is the sum of all the inscribed rectangles areas

$$A_a^b \approx \sum_{k=1}^{n} [f(x_k) - g(x_k)] \Delta x$$

*this is the Riemann sum

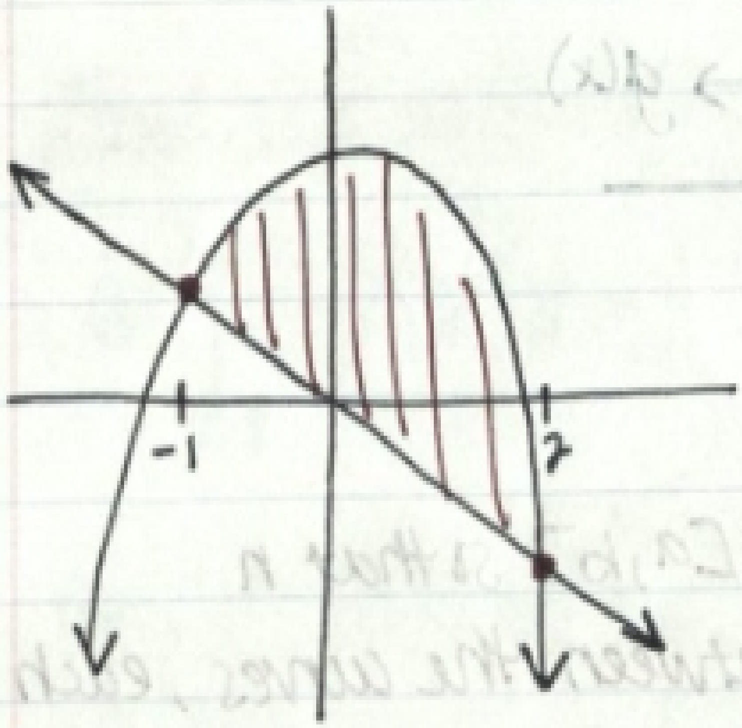
3. the actual area between the curves over [a, b]

$$A_a^b = \lim_{n \to \infty} \sum_{k=1}^{n} [f(x_k) - g(x_k)] \Delta x$$

$$\therefore A_a^b = \int_a^b [f(x) - g(x)]\, dx$$ By definition of a definite integral

Note: $f(x) - g(x)$ = top function minus the bottom one

Example: Find the area enclosed by $y = -x$ and $y = 2 - x^2$



$$-x = 2 - x^2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \text{ or } x = -1$$

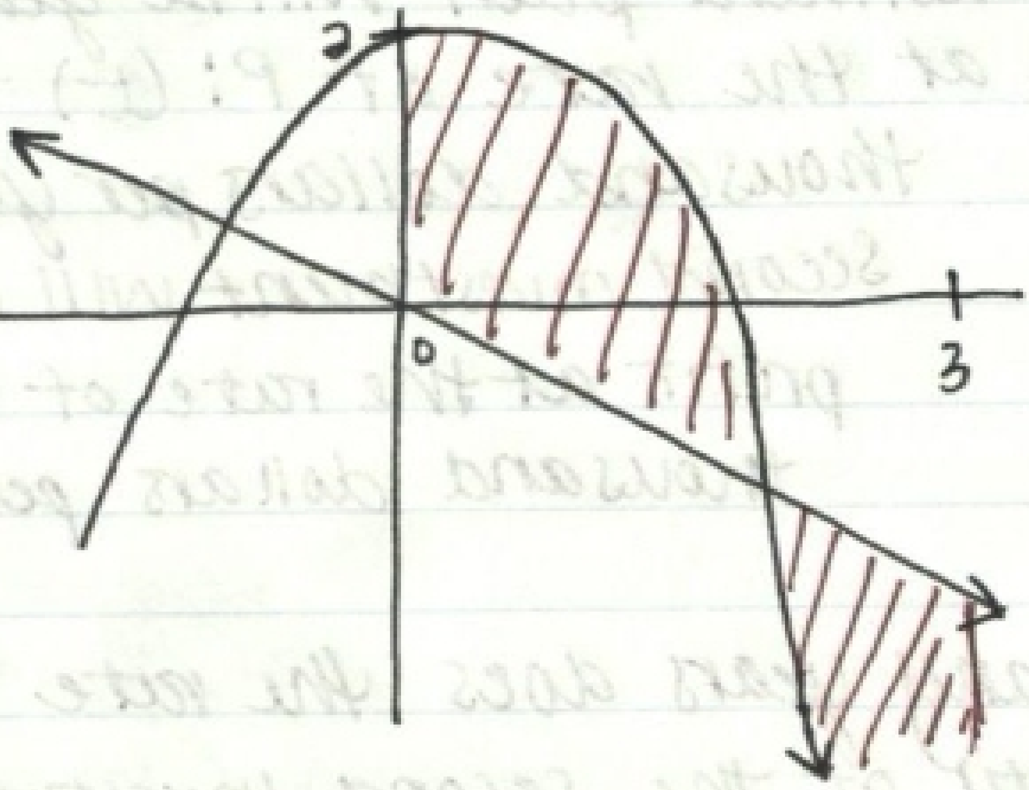
Limits!

$$A = \int_{-1}^{2} \left[(2-x^2) - (-x)\right] dx$$

$$A = \int_{-1}^{2} (2 + x - x^2)\, dx$$

$$A = \left[2x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-1}^{2} = \boxed{\frac{9}{2} \text{ units}^2}$$

Example: Find the area between $y = -x$ and $y = 2 - x^2$ from $x = 0$ to $x = 3$



$$A = \int_0^2 [2 - x^2 - (-x)]\,dx + \int_2^3 [-x - (2 - x^2)\,dx]$$

$$A = \int_0^2 (2 + x - x^2)\,dx + \int_2^3 [-x - 2 + x^2]$$

$$A = \left[2x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^2 + \left[-2x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right]_2^3$$

$$A = \left(3\frac{1}{3} - 0\right) + \left(\underbrace{-1\frac{1}{2} + 3\frac{1}{3}}_{1\,5/6}\right)$$

$$\boxed{A = 5\frac{1}{6} \text{ units}}$$