

## McNemar's Test

Previous test only valid if the two samples are independent; for example, students in SPHTM vs. students in SOM. Sometimes, it is obvious they are not independent. Perhaps we have matched pairs or perhaps each subject is his/her own control. The outcome of interest is categorical, however. We might want to test whether two skin lotions are equally effective at relieving a poison ivy rash. We test the two lotions on 100 persons with a poison ivy rash on each arm, applying lotion one to one arm and lotion two to the second arm.

Let's construct a different kind of 2 by 2 table. Let the matched pair be the unit of analysis and classify by the occurrence of the event or variable of interest for each member of the pair. Pairs fall into two groups. A concordant pair is one in which the event is the same for each member of the pair. A discordant pair is one in which the event is different for the two members of the pair.

	Other member of pair		Total
One member of pair	+	-	
+		$n_a$	
-	$n_b$		
Total			

$$n_D = n_a + n_b$$

**Note: Concordant pairs provide no information about differences between the two groups in the occurrence of the event. We focus on the discordant pairs only in the analysis.**

**Type A discordant pairs. First member of the pair has the event and the second does not.**

**Type B discordant pairs. Second member of the pair has the event and the first does not.**

**Let  $p$  = probability that a discordant pair is of type A. If the event of interest occurs equally in the two groups, we expect  $p$  to be 0.5.**

$$H_0: p = 0.5$$

$$H_1: p \neq 0.5$$

**Two analytical approaches. Normal-Theory test vs. an exact test. We will focus first on the normal theory test.**

**Suppose there are  $n_D$  discordant pairs,  $n_A$  of type A. Under  $H_0$ ,  $E(n_A) = n_D / 2$  and  $\text{Var}(n_A) = n_D/4$ . These come from the mean and variance of a binomial distribution. Assume the normal approximation to the binomial distribution is appropriate, and we will include a correction for continuity.**

**This approach is valid if  $npq = n_D / 4 \geq 5$  or  $n_D > 20$ .**