Problem 12.1 The value of π is 3.1415962654.... If C is the circumference of a circle and r is its radius, determine the value of r/C to four significant digits.

Solution:

$$C = 2\pi r \Rightarrow \frac{r}{C} = \frac{1}{2\pi} = 0.159154943.$$

To four significant digits we have $\boxed{\frac{r}{C} = 0.1592}$

Problem 12.2 The base of natural logarithms is $e = 2.718281828\ldots$

(a) Express e to five significant digits.
(b) Determine the value of e^2 to five significant digits.
(c) Use the value of e you obtained in part (a) to determine the value of e^2 to five significant digits.

[Part (c) demonstrates the hazard of using rounded-off values in calculations.]

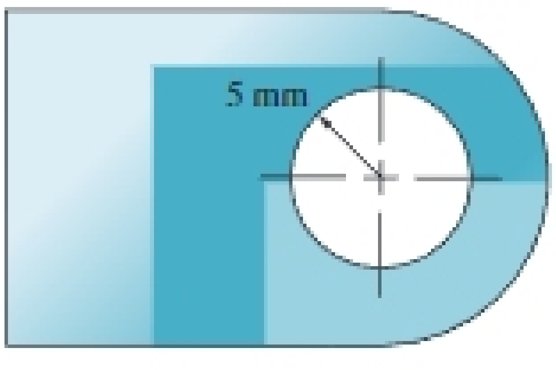
Solution: The value of e is: $e = 2.718281828$

(a) To five significant figures $\boxed{e = 2.7183}$

(b) e^2 to five significant figures is $\boxed{e^2 = 7.3891}$

(c) Using the value from part (a) we find $\boxed{e^2 = 7.3892}$ which is not correct in the fifth digit.

Problem 12.3 A machinist drills a circular hole in a panel with a nominal radius $r = 5$ mm. The actual radius of the hole is in the range $r = 5 \pm 0.01$ mm. (a) To what number of significant digits can you express the radius? (b) To what number of significant digits can you express the area of the hole?



Solution:

(a) The radius is in the range $r_1 = 4.99$ mm to $r_2 = 5.01$ mm. These numbers are not equal at the level of three significant digits, but they are equal if they are rounded off to two significant digits.

$\boxed{\text{Two: } r = 5.0 \text{ mm}}$

(b) The area of the hole is in the range from $A_1 = \pi r_1^2 = 78.226$ m^2 to $A_2 = \pi r_2^2 = 78.854$ m^2. These numbers are equal only if rounded to one significant digit:

$\boxed{\text{One: } A = 80 \text{ mm}^2}$

Problem 12.4 The opening in the soccer goal is 25 ft wide and 8 ft high, so its area is 24 ft × 8 ft = 192 ft^2. What is its area in m^2 to three significant digits?

Solution:

$$A = 192 \text{ ft}^2 \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)^2 = 17.8 \text{ m}^2$$

$\boxed{A = 17.8 \text{ m}^2}$

Problem 12.5 The Burj Dubai, scheduled for completion in 2008, will be the world's tallest building with a height of 705 m. The area of its ground footprint will be 8000 m^2. Convert its height and footprint area to U.S. customary units to three significant digits.

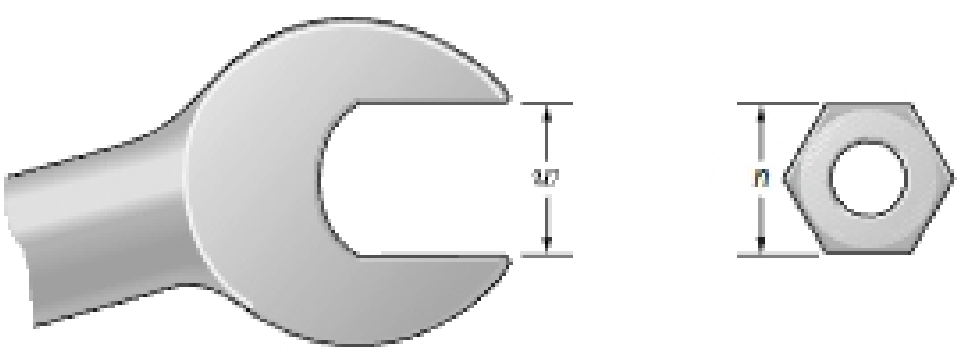
Solution:

$$h = 705 \text{ m}\left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = 2.31 \times 10^3 \text{ ft}$$

$$A = 8000 \text{ m}^2\left(\frac{3.218 \text{ ft}}{1 \text{ m}}\right)^2 = 8.61 \times 10^4 \text{ ft}^2$$

$$\boxed{h = 2.31 \times 10^3 \text{ ft}, \quad A = 8.61 \times 10^4 \text{ ft}^2}$$

Problem 12.6 Suppose that you have just purchased a Ferrari F355 coupe and you want to know whether you can use your set of SAE (U.S. Customary Units) wrenches to work on it. You have wrenches with widths $w = 1/4$ in, 1/2 in, 3/4 in, and 1 in, and the car has nuts with dimensions $n = 5$ mm, 10 mm, 15 mm, 20 mm, and 25 mm. Defining a wrench to fit if w is no more than 2% larger than n, which of your wrenches can you use?



Solution: Convert the metric size n to inches, and compute the percentage difference between the metric sized nut and the SAE wrench. The results are:

$$5 \text{ mm}\left(\frac{1 \text{ inch}}{25.4 \text{ mm}}\right) = 0.19685.. \text{ in}, \quad \left(\frac{0.19685 - 0.25}{0.19685}\right)100 = -27.0\%$$

$$10 \text{ mm}\left(\frac{1 \text{ inch}}{25.4 \text{ mm}}\right) = 0.3937.. \text{ in}, \quad \left(\frac{0.3937 - 0.5}{0.3937}\right)100 = -27.0\%$$

$$15 \text{ mm}\left(\frac{1 \text{ inch}}{25.4 \text{ mm}}\right) = 0.5905.. \text{ in}, \quad \left(\frac{0.5905 - 0.5}{0.5905}\right)100 = +15.3\%$$

$$20 \text{ mm}\left(\frac{1 \text{ inch}}{25.4 \text{ mm}}\right) = 0.7874.. \text{ in}, \quad \left(\frac{0.7874 - 0.75}{0.7874}\right)100 = +4.7\%$$

$$25 \text{ mm}\left(\frac{1 \text{ inch}}{25.4 \text{ mm}}\right) = 0.9843.. \text{ in}, \quad \left(\frac{0.9843 - 1.0}{0.9843}\right)100 = -1.6\%$$

A negative percentage implies that the metric nut is smaller than the SAE wrench; a positive percentage means that the nut is larger then the wrench. Thus within the definition of the 2% fit, the 1 in wrench will fit the 25 mm nut. **The other wrenches cannot be used.**

Problem 12.7 Suppose that the height of Mt. Everest is known to be between 29,032 ft and 29,034 ft. Based on this information, to how many significant digits can you express the height (a) in feet? (b) in meters?

Solution:

(a) $h_1 = 29032$ ft

$h_2 = 29034$ ft

The two heights are equal if rounded off to four significant digits. The fifth digit is not meaningful.

$\boxed{\text{Four: } h = 29{,}030 \text{ ft}}$

(b) In meters we have

$$h_1 = 29032 \text{ ft}\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 8848.52 \text{ m}$$

$$h_2 = 29034 \text{ ft}\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 8849.13 \text{ m}$$

These two heights are equal if rounded off to three significant digits. The fourth digit is not meaningful.

$\boxed{\text{Three: } h = 8850 \text{ m}}$

Problem 12.8 The maglev (magnetic levitation) train from Shanghai to the airport at Pudong reaches a speed of 430 km/h. Determine its speed (a) in mi/h; (b) ft/s.

Solution:

(a) $v = 430\dfrac{\text{km}}{\text{h}}\left(\dfrac{0.6214 \text{ mi}}{1 \text{ km}}\right) = 267 \text{ mi/h}$ $\boxed{v = 267 \text{ mi/h}}$

(b) $v = 430\dfrac{\text{km}}{\text{h}}\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ ft}}{0.3048 \text{ m}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) = 392 \text{ ft/s}$

$\boxed{v = 392 \text{ ft/s}}$

Problem 12.9 In the 2006 Winter Olympics, the men's 15-km cross-country skiing race was won by Andrus Veerpalu of Estonia in a time of 38 minutes, 1.3 seconds. Determine his average speed (the distance traveled divided by the time required) to three significant digits (a) in km/h; (b) in mi/h.

Solution:

(a) $v = \dfrac{15 \text{ km}}{\left(38 + \dfrac{1.3}{60}\right)\text{min}}\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right) = 23.7 \text{ km/h}$ $\boxed{v = 23.7 \text{ km/h}}$

(b) $v = (23.7 \text{ km/h})\left(\dfrac{1 \text{ mi}}{1.609 \text{ km}}\right) = 14.7 \text{ mi/h}$ $\boxed{v = 14.7 \text{ mi/h}}$