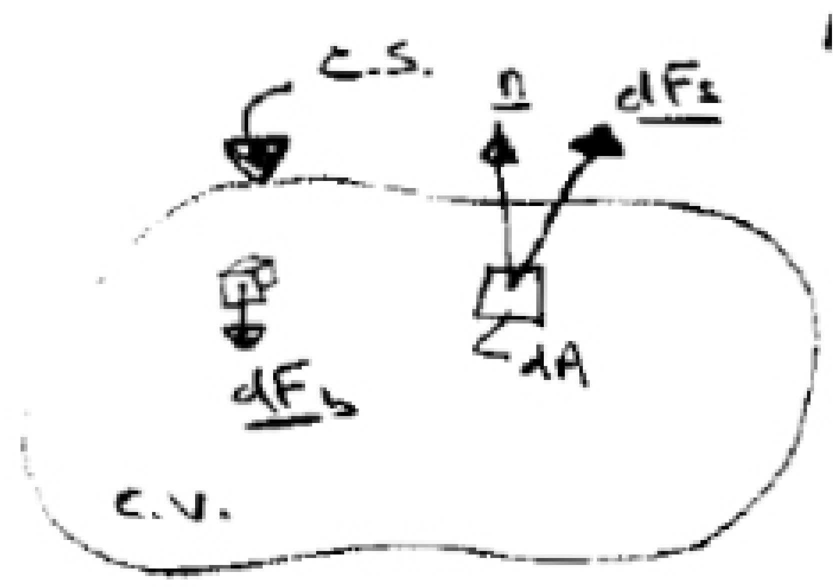
Chapter 2: Pressure Distribution in a Fluid

Pressure and pressure gradient

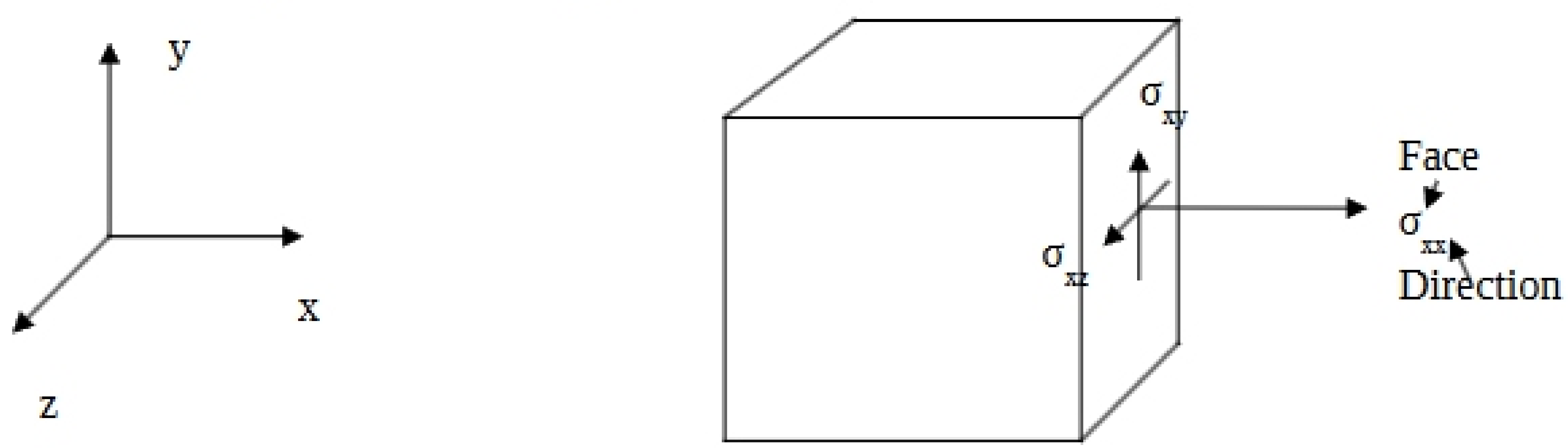


In fluid statics, as well as in fluid dynamics, the forces acting on a portion of fluid (C.V.) bounded by a C.S. are of two kinds: body forces and surface forces.

Body Forces: act on the entire body of the fluid (force per unit volume).

Surface Forces: act at the C.S. and are due to the surrounding medium (force/unit area-stress).

In general the surface forces can be resolved into two components: one normal and one tangential to the surface. Considering a cubical fluid element, we see that the stress in a moving fluid comprises a 2nd order tensor.

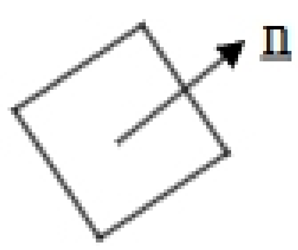


$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Since, by definition, a fluid cannot withstand a shear stress without moving, (deformation) a stationary fluid must necessarily be completely free of shear stress ($\sigma_{ij}=0$, $i \neq j$). The only non-zero stress is the normal stress, which is referred to as pressure:

$$\sigma_{ii} = -p$$

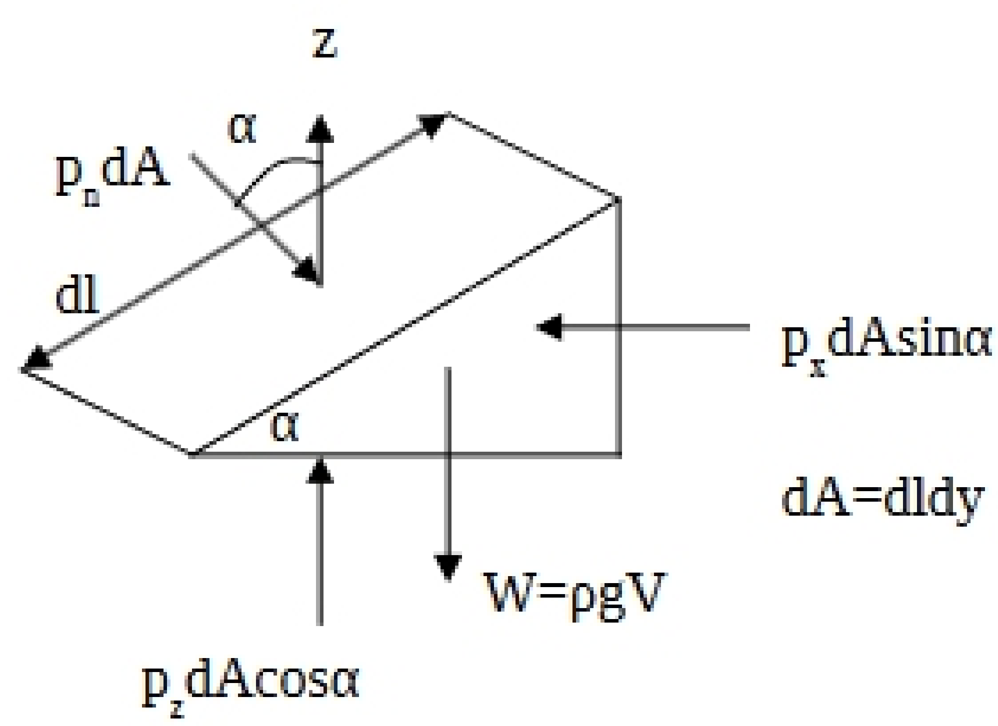
$\sigma_n = -p$, which is compressive, as it should be since fluid cannot withstand tension. (Sign convention based on the fact that p>0 and in the direction of $-\underline{n}$)



Or $p_x = p_y = p_z = p_n = p$

(one value at a point, independent of direction, p is a scalar)

i.e. normal stress (pressure) is isotropic. This can be easily seen by considering the equilibrium of a wedge shaped fluid element



Where

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Note: For a fluid in motion, the normal stress is different on each face and not equal to p.

$$\sigma_{xx} \neq \sigma_{yy} \neq \sigma_{zz} \neq -p$$

By convention p is defined as the average of the normal stresses

$$p = -\frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right) = -\frac{1}{3}\sigma_{ii}$$

The fluid element experiences a force on it as a result of the fluid pressure distribution if it varies spatially. Consider the net force in the x direction due to p(x,t).

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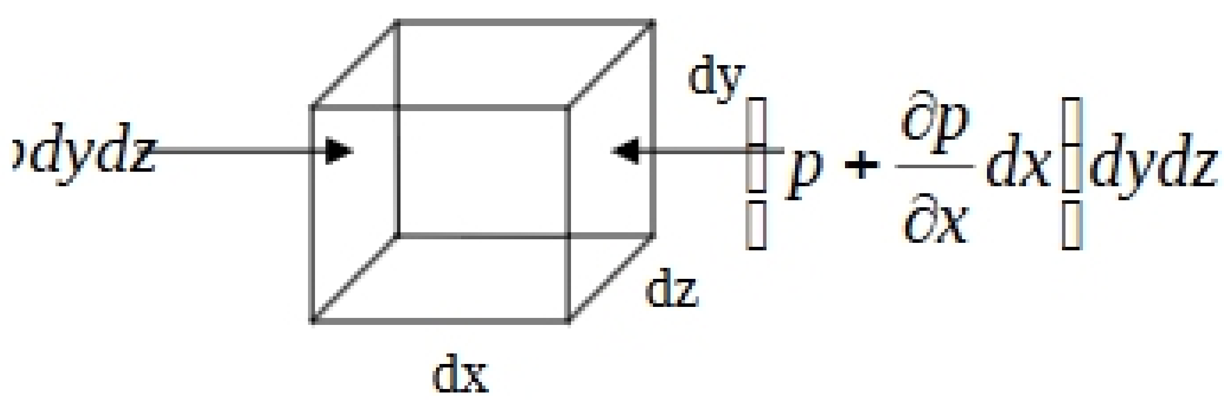

The result will be similar for dF_y and dF_z; consequently, we conclude:

$$d\underline{F}_{press} = \left(-\frac{\partial p}{\partial x}\hat{i} - \frac{\partial p}{\partial y}\hat{j} - \frac{\partial p}{\partial z}\hat{k}\right) d\forall$$

Or: $\underline{f} = -\nabla p$ force per unit volume due to p(x,t).

Note: if p=constant, $\underline{f} = 0$.