

Section 2.2: Systems of Linear Equations: Unique Solutions**Section 2.3: Systems of Linear Equations: Underdetermined and Overdetermined Systems****Solving System of Equations.**

Definition: An **augmented matrix** is a condensed method of representing a system of equations.

Example: Represent the system of equations as an augmented matrix.

$$3x + 2y = 7$$

$$x + 4y = 10$$

Example: Give the system of equations represented by the augmented matrix. (Variables are listed in the first row.)

$$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & -4 & 2 & 10 \\ 3 & 4 & 2 & 20 \end{array} \right]$$

Row Operations are used to manipulate an augmented matrix into a form (usually row reduced form) where the solution can easily be discerned. The three row operations are:

- 1) Swap Rows
- 2) Multiply a row by a non-zero number
- 3) Add a multiple of one row to another row.

Row Reduced Form (reduced row echelon form)

1. The first non-zero number in a row is a 1 (called a leading one).
2. The leading one is the only non-zero number in a column.
3. The leading ones are in a diagonal like fashion from the upper left to the lower right.

Example: Which of these matrices are in row reduced form?

A) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array} \right]$

B) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 2 & 7 \end{array} \right]$

C) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & 6 \end{array} \right]$

D) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 10 \end{array} \right]$

Example: Solve the system of equations.

$$3x + 2y = 7$$

$$x + 4y = 10$$

Example: Create a leading one in the row one column one position.

$$3x + y + 2z = 11$$

$$4x + 9y + z = 25$$

$$2x - y + 3z = 9$$

Example: Solve these system of equations.

$$\begin{aligned} \text{A)} \quad & 3x + y - 9 = 0 \\ & x - y + z - 4 = 0 \\ & 3x + z - 11 = 0 \\ & 4x - y + 2z = 15 \end{aligned}$$

$$\begin{aligned} \text{B)} \quad & x + y - 3z = 0 \\ & 2x - 3y + z = 1 \\ & 4x - y - 5z = 1 \end{aligned}$$

$$\begin{aligned} \text{C)} \quad & x + 3y - z - 3w = 7 \\ & 2x + 4y - 2w = 10 \end{aligned}$$