Binomial Distribution

n **independent Bernoulli trials—each with only two possible outcomes**

Probability of success at each trial is some constant, *p*

Probability of failure = *q* = 1 – *p*

Example: In throwing a fair die, let a "success" be a 6 on the top face.

Therefore, *p* = 1/6

$$q = 1 - p = 1 - 1/6 = 5/6$$

In 5 trials, what is the probability of 5 successes?

$$p^5 = p*p*p*p*p = \left(\frac{1}{6}\right)^5 = \frac{1}{7776} = 0.0001286$$

In 5 trials, what is the probability of 5 failures?

$$q^5 = \left(\frac{5}{6}\right)^5 = 0.40188$$

What is the probability of 2 successes and 3 failures?

SSFFF	**FSSFF**	**FFSFS**
SFSFF	**FSFSF**	**FFFSS**
SFFSF	**FSFFS**	
SFFFS	**FFSSF**	

This is the number of ways of selecting 2 outcomes out of 5 trials—order doesn't matter

$$\binom{5}{2} = \frac{5!}{3!\,2!} = 10$$

Therefore, the probability of 2 successes and 3 failures in 5 trials is

$$\binom{5}{2} p^2 q^3$$

$$= 10 * \left(\frac{1}{6}\right)^2 * \left(\frac{5}{6}\right)^3$$

$$= 10 * \frac{1}{36} * \frac{125}{216}$$

$$= 0.16075$$

Generalize: Probability of *k* successes in n trials (order does not matter) is

$$\Pr(X = k) = \frac{n!}{(n-k)!k!} p^k q^{n-k}$$

where *k* = 0, 1, 2, … , *n*

and *q* = 1 – *p*

The distribution is the number of successes in n statistically independent trials, where the probability of success on each trial = *p*, is called the <u>Binomial Distribution</u>. The probability mass function is

$$\Pr(X = k) = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, 2, \ldots, n$$