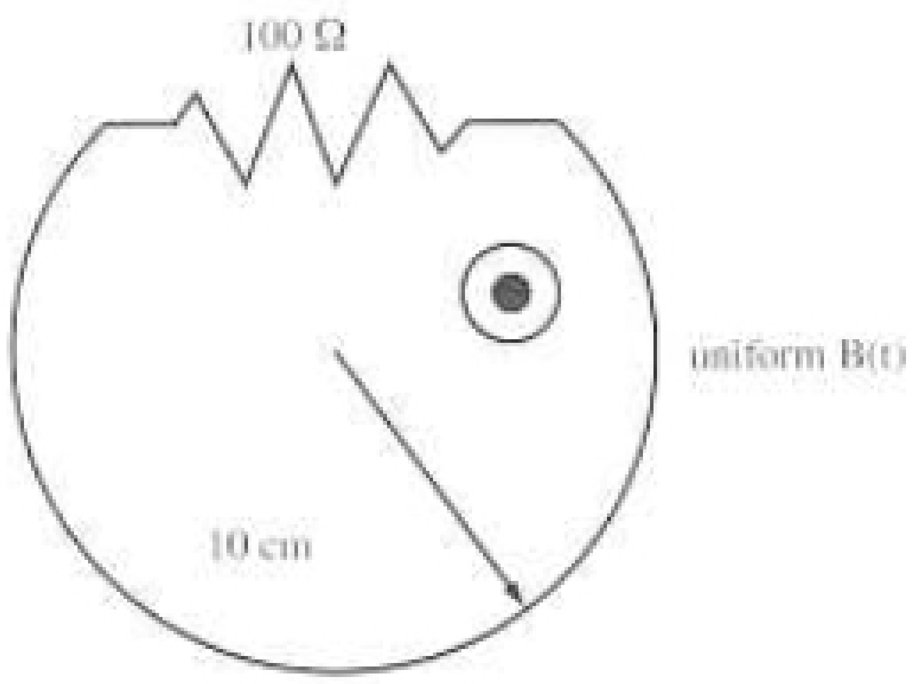
5.8 Problems

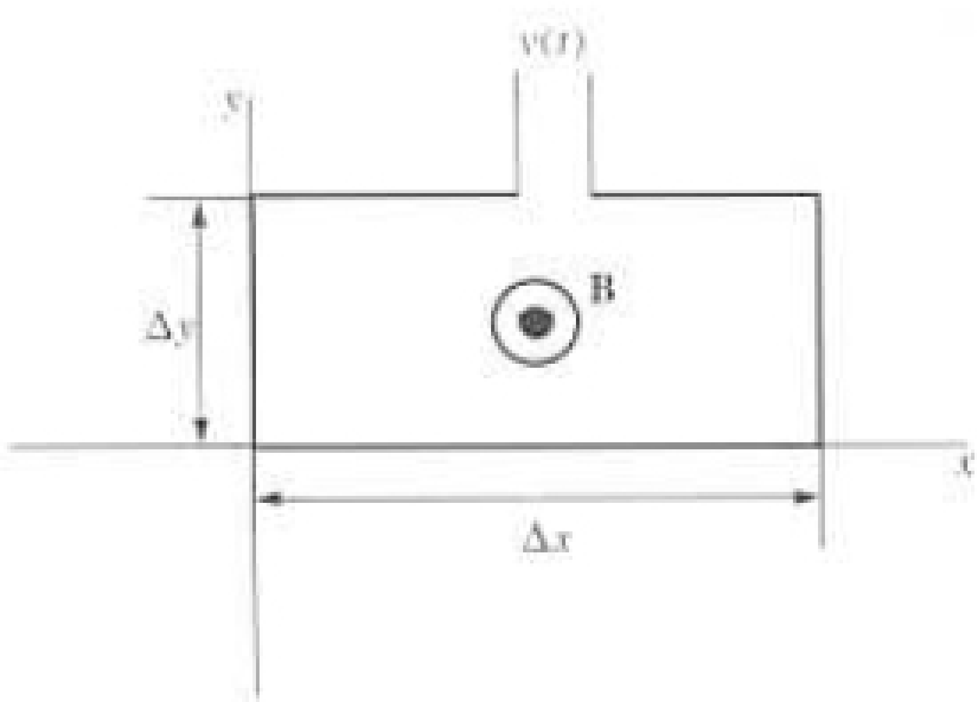
5.1.1. In a source-free region, we find that $\mathbf{B} = z\mathbf{u}_y + x\mathbf{u}_z$. Does **E** vary with time?

5.1.2. A perfect conductor joins two ends of a 100 Ω resistor, and the closed loop is in a region of uniform magnetic flux density $B = 10\, e^{(-t/10)}$ T.



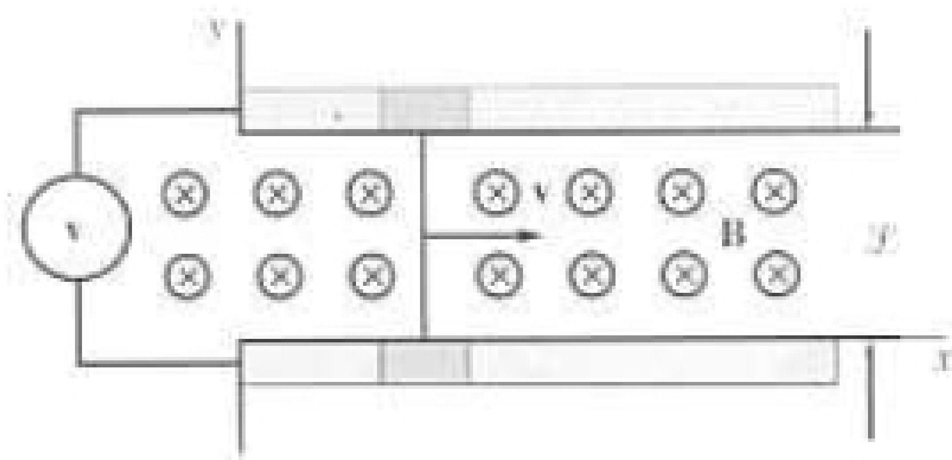
Neglecting the self-inductance of the loop, find and plot the voltage $V(t)$ that appears across the 100 Ω resistor. A device based on this principle is used to monitor time-varying magnetic fields in experiments and in biological studies.

5.1.3. A closed loop ($\Delta x = 30$ cm × $\Delta y = 20$ cm) of wire passes through a nonuniform time-independent magnetic field $\mathbf{B} = y\mathbf{u}_z$ T with a constant velocity $\mathbf{v}_0 = 5\mathbf{u}_x$ m/s. At $t = 0$, the loop's lower left corner is located at the origin. Find an expression for the voltage V_t generated by the loop as a function of time. You may neglect the magnetic field created by the current in the loop.



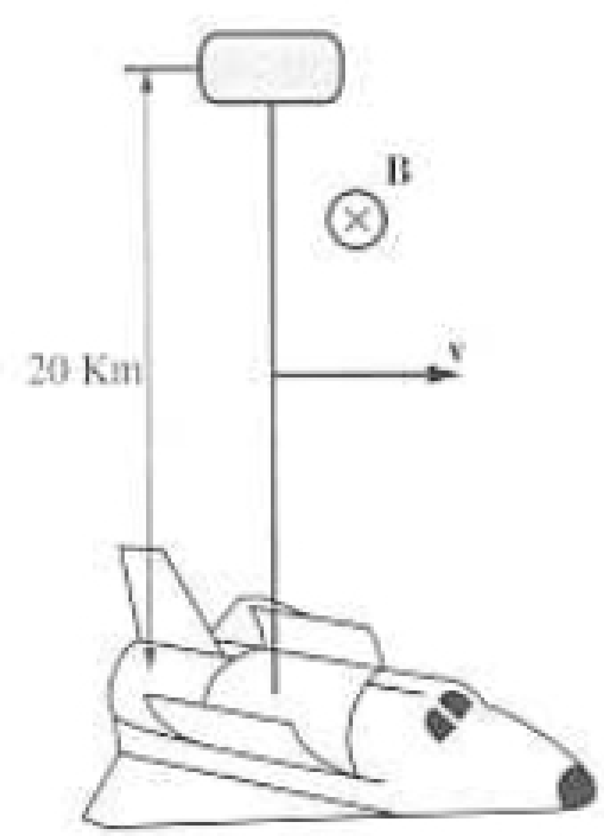
5.1.4. Repeat Problem 5.1.3 with the magnetic flux density being uniform in space $\mathbf{B} = 0.1\mathbf{u}_z$ T. Explain your result.

5.1.5. Find the generated voltage if the axle moves at a constant velocity $\mathbf{v} = v\mathbf{u}_x = 3\mathbf{u}_x$ m/s in a uniform magnetic field of $\mathbf{B} = B_0\mathbf{u}_z = 5\mathbf{u}_z$ T. At $t = 0$, the axle was at $x = 0$, $L = 40$ cm.

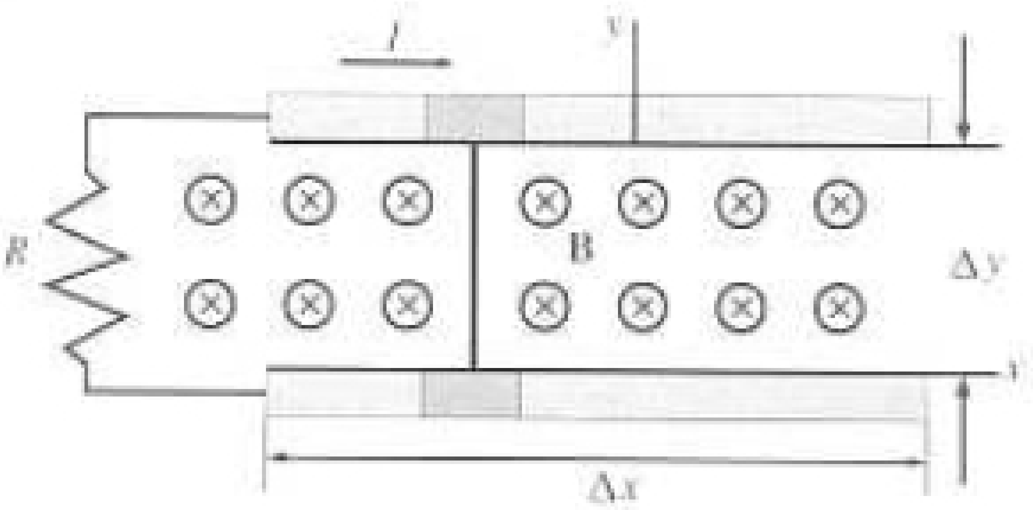


5.1.6. Repeat Problem 5.1.5 with the constraint that the rails separate with $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1 x$. The wheels are free to slide on the "trombone-like" axle so they remain on the rails ($\mathscr{L}_0 = 0.4$ m, $\mathscr{L}_1 = 0.04$ m).

5.1.7. A tethered satellite is to be connected to the shuttle to generate electricity as it passes through the ambient plasma. A plasma consists of a large number of positive charges and negative charges. Assuming that the shuttle takes two hours to go around the Earth, find the expected voltage difference ΔV between the tether and the shuttle. The shuttle flies approximately 400 km above the earth where $\mathbf{B} = 10^{-5}$ T.

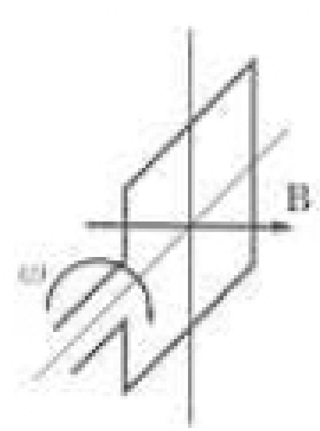


5.1.8. A conducting axle oscillates over two conducting parallel rails in a uniform magnetic field $\mathbf{B} = B_0\mathbf{u}_z$ ($B_0 = 4$ T). The position of the axle is given by $x = (\Delta x/2)\,[1 - \cos\omega t]$ ($\Delta x = 0.2$ m, $\omega = 500\ \mathrm{s}^{-1}$). Find and plot the current $I(t)$ if the resistance is $R = 10\ \Omega$ and the distance between the rails is $\Delta y = 0.1$ m.

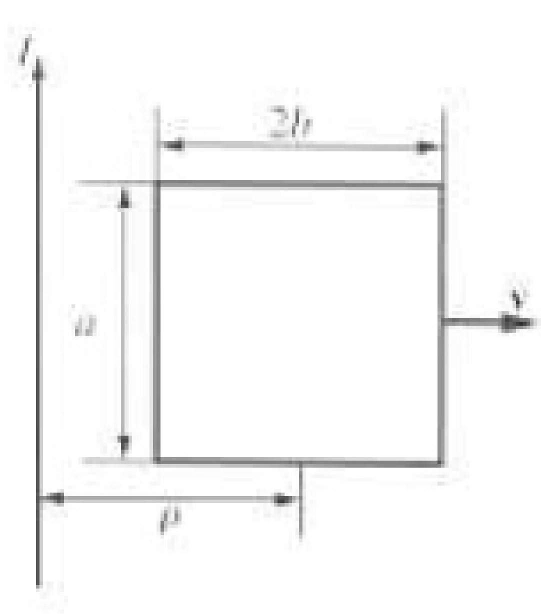


5.1.9. Repeat Problem 5.1.8 with the magnetic field also varying in time as $\mathbf{B} = B_0\cos\omega t\,\mathbf{u}_z$ with B_0 and ω having the same values.

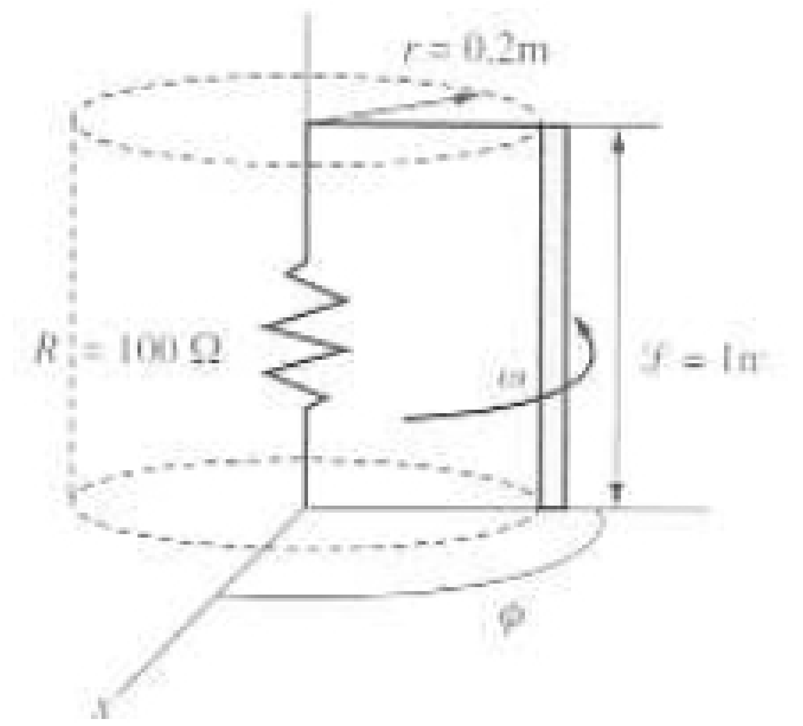
5.1.10. Calculate the voltage that is induced between the two nodes as the coil with dimensions 0.5 m × 0.5 m rotates in a uniform magnetic field with a flux density $B = 2$ T with a constant angular frequency $\omega = 1200\ \mathrm{s}^{-1}$.



5.1.11. A square loop is adjacent to an infinite wire that carries a current I. The loop moves with a velocity $\mathbf{v} = v_0\mathbf{u}_\rho$. The center of the loop is at ρ, and the initial position is $\rho = b$. Determine the induced voltage $V(t)$ in the loop assuming dimensions $a \times 2b$.



5.1.12. The 1 m long wire shown in the figure rotates with an angular frequency $\omega = 40\pi\ \mathrm{s}^{-1}$ in the magnetic field $\mathbf{B} = 0.5\cos\phi\,\mathbf{u}_\phi$ T. Find the current in the closed loop with a resistance 100 Ω.



5.2.1. The current density is $\mathbf{J} = \sin(\pi x)\mathbf{u}_x$. Find the time rate of increase of the charge density $\partial\rho_v/\partial t$ at $x = 1$.

5.2.2. The current density is $\mathbf{J} = e^{(-\rho^2)}\mathbf{u}_\rho$ in cylindrical coordinates. Find the time rate of increase of the charge density at $\rho = 1$.

5.3.1. Compare the magnitudes of the conduction and displacement current densities in copper ($\sigma = 5.8 \times 10^7$ S/m, $\varepsilon = \varepsilon_0$), sea water ($\sigma = 4$ S/m, $\varepsilon = 81\,\varepsilon_0$), and earth ($\sigma = 10^{-3}$ S/m, $\varepsilon = 10\,\varepsilon_0$) at 60 Hz, 1 MHz, and at 1 GHz.

5.3.2. Given the conduction current density in a lossy dielectric as $J_c = 0.2\sin(2\pi 10^9 t)$ A/m², find the displacement current density if $\sigma = 10^{-3}$ S/m and $\varepsilon_r = 6.5$.

5.4.1. Show that the fields $\mathbf{B} = B_0\cos\omega t\,\mathbf{u}_x$ and $\mathbf{E} = E_0\cos\omega t\,\mathbf{u}_z$ do not satisfy Maxwell's equations in air $\varepsilon_r = 1$. Show that the fields $\mathbf{B} = B_0\cos(\omega t - ky)\mathbf{u}_x$ and $\mathbf{E} = E_0\cos(\omega t - ky)\mathbf{u}_z$ satisfy these equations. What is the value of k in terms of the other stated parameters?

5.4.2. Given

$$\mathbf{E} = E_0\cos(\omega t - ky)\mathbf{u}_z$$

$$\text{and}\quad \mathbf{H} = \left(\frac{E_0}{Z_0}\right)\cos(\omega t - ky)\mathbf{u}_x$$

in a vacuum, find Z_0 in terms of ε_0 and μ_0 so Maxwell's equations are satisfied.

5.4.3. Do the fields

$$\mathbf{E} = E_0 \cos x \cos t \mathbf{u}_y \text{ and } \mathbf{H} = \left(\frac{E_0}{\mu_0}\right) \sin x \sin t \mathbf{u}_z$$

satisfy Maxwell's equations?

5.4.4. Find a charge density ρ_v that could produce an electric field in a vacuum $\mathbf{E} = E_0 \cos x \cos t \mathbf{u}_x$.

5.4.5. Find the displacement current density flowing through the dielectric of a coaxial cable of radii a and b where $b > a$ if a voltage $V_0 \cos \omega t$ is connected between the two conducting cylinders.

5.4.6. Find the displacement current density flowing through the dielectric of two concentric spheres of radii a and b where $b > a$ if a voltage $V_0 \cos \omega t$ is connected between the two conducting spheres.

5.4.7. Starting from Maxwell's equations, derive the equation of continuity.

5.4.8. Write all of the terms that appear in Maxwell's equations in Cartesian coordinates.

5.5.1. If $\mathbf{E} = E_0 \cos(\omega t - \beta z)\mathbf{u}_x$ is a solution to Maxwell's equations, find $\mathbf{H}$. Find $\mathbf{S}_{av}$.

5.5.2. If $\mathbf{H} = H_0 \cos(\omega t - \beta z)\mathbf{u}_x$ is a solution to Maxwell's equations, find $\mathbf{E}$. Find $\mathbf{S}_{av}$.

5.5.3. Compute the stored electric energy that is stored in a cube whose volume is 1 m^3 in which a uniform electric field of 10^4 V/m exists. Compute the stored energies if the cube is empty and if it is filled with water that has $\varepsilon = 81\varepsilon_0$.

5.6.1. Write $\mathbf{E} = 120\pi \cos(3 \times 10^9 t - 10z)\mathbf{u}_x$ and $\mathbf{H} = 1 \cos(3 \times 10^9 t - 10z)\mathbf{u}_y$ in phasor notation.

5.6.2. Write the phasors $\mathbf{E} = 3e^{-j\beta z}\mathbf{u}_x$ and $\mathbf{H} = 0.4e^{-j45^\circ} e^{-j\beta z}\mathbf{u}_y$ in the time domain. The frequency of oscillation is ω. Find the average Poynting vector $\mathbf{S}_{av}$.

5.6.3. At a frequency of $f = 1$ MHz, verify that copper ($\sigma = 5.8 \times 10^7$ S/m, $\varepsilon_r = 1$) is a good conductor, and quartz ($\sigma = 10^{-17}$ S/m, $\varepsilon_r = 4$) is a good insulator.

5.6.4. Find the frequency where quartz becomes a conductor.

5.6.5. Find the frequency where copper becomes an insulator.