

Section 7.5: Conditional Probability and Independence and Section 7.6: Bayes' Theorem

Definition: Conditional probability is the probability of an event, A, will occur given that another event, B, has already happened. This is denoted as $P(A|B)$.

Example: This table classifies the English, History, Math, and Poly Sci majors at State U according to their year. (There are no double majors.)

| | Freshmen(F) | Sophomores(Soph) | Juniors(J) | Seniors(Sr) | Totals |
|--------------|-------------|------------------|------------|-------------|--------|
| English(E) | 64 | 35 | 31 | 41 | 171 |
| History(H) | 55 | 41 | 33 | 52 | 181 |
| Math(M) | 29 | 32 | 50 | 69 | 180 |
| Poly Sci(PS) | 70 | 33 | 41 | 37 | 181 |
| Totals | 218 | 141 | 155 | 199 | 713 |

If a student is selected at random, find

A) $P(H | \text{soph}) =$

B) $P(\text{soph} \cup J | \text{PS}) =$

Interpret these Statements: Express these questions using the correct probability notation.

A factory makes widgets on three different machines; A, B and C; and a widget is selected at random.

1. What is the probability that it is defective knowing that the widget was made on machine C?
2. If the widget was made on machine A, what is the probability that it is defective?
3. What is the probability it is made on machine A and is not defective?
4. What is the probability the widget is selected from line B or is defective?
5. What is the probability that the defective widget was made on machine B?
6. What is the probability that the widget is defective and came from machine A or machine B?

Example: Use the probability distribution and the events to answer these questions.

| S | a | b | c | d | e | f |
|------|------|------|------|------|------|------|
| prob | 0.15 | 0.08 | 0.21 | 0.12 | 0.25 | 0.19 |

$$E = \{a, c, d, e\} \quad F = \{b, d, f\} \quad G = \{a, b, d\}$$

Compute

A) $P(E|G) =$

B) $P(G|F) =$

Definition: Let E and F be two events of the sample space S . Then the conditional probability that E occurs given that F has occurred is defined as

$$P(E|F) =$$

Example: It is known from a survey that 29% of the people buy product A, 36% of the people buy product B and 11% buy both products. Find the probability that

A) the person buys product A if they bought product B.

B) the person does not buy product B knowing that they bought product A.

Example: A Jar has 5 red, 4 green, and 3 yellow items. Two items are drawn in succession without replacement. Construct the probability tree that represents this experiment.

Example: Draw the tree that represents this experiment: Draw one ball from box A and place it into box B. Then draw one ball from box B.

| | |
|--------------|--------------|
| <u>Box A</u> | <u>Box A</u> |
| 3 red | 2 red |
| 5 green | 5 yellow |