One-Sample Test
Two-Sided Alternative

Example: In a cardiology study, one variable of interest is Body Mass Index (BMI) which is measured as weight (kg)/height^2 (m^2). A total of 14 adult males were studied. The data are given in the table below. Can we conclude that the mean BMI of the population from which the sample was drawn is 35?

Subject	BMI	Subject	BMI	Subject	BMI
1	23	6	21	11	23
2	25	7	23	12	26
3	21	8	24	13	31
4	37	9	32	14	45
5	39	10	57		

$$n = 14, \mu_o = 35, \overline{X} = 30.5, s = 10.6392$$

H_o: $\mu = 35$
H_1: $\mu \neq 35$

Let $\alpha = 0.05$

Def: A <u>two-tailed</u> or <u>two-sided</u> test is one in which the parameter under study in the alternative hypothesis can be less than or greater than the value of the parameter in the null hypothesis.

Reject H_o if $t > c_2$ or $t < c_1$
Accept H_o if $c_1 \leq t \leq c_2$

To determine c_1, c_2 we need to specify α.

Pr (reject H_o | H_o true) = Pr ($t < c_1$ | H_o true) + Pr ($t > c_2$ | H_o true)

$$= \alpha/2 + \alpha/2 = \alpha$$

$t \sim t_{n-1}$ under H_o

Therefore, $c_1 = t_{n-1, \alpha/2}$ and $c_2 = t_{n-1, 1-\alpha/2}$ are the upper and lower critical values.

Therefore, reject H_o if

$$t > = t_{n-1, 1-\alpha/2} \text{ or if } t < t_{n-1, \alpha/2}$$

Accept H_o if $t_{n-1, \alpha/2} \leq t \leq t_{n-1, 1-\alpha/2}$
Example: $\alpha = 0.05$

$t_{n\text{-}1,\ 1\text{-}\alpha/2} = t_{13,\ 0.975} = 2.160$

$t_{n\text{-}1,\ \alpha/2} = t_{13,\ 0.025} = -2.160$

Reject H_o if $t > 2.160$ or if $t < -2.160$

$$t = \frac{\overline{X} - \mu_o}{s/\sqrt{n}} = \frac{30.5 - 35}{10.6392/\sqrt{14}} = \frac{-4.5}{2.8434} = -1.58$$

Decision: Do not reject H_o.

Conclusion: We have insufficient evidence to state that the mean BMI of the population from which the sample came is anything other than 35.

p-value: Probability of getting a value of the test statistic this extreme or more extreme if H_o is true.

a. **If $t \leq 0$, $p = 2 * \Pr(t_{n-1} \leq t)$**

b. **If $t > 0$, $p = 2 * [1 - \Pr(t_{n-1} \leq t)]$**

For the example, this is 2 * the area to the left of the test statistic, t, with 13 df.

$t_{13,\ 0.05} = -1.771$

$t = -1.58$

$t_{13,\ 0.10} = -1.350$

Therefore, $0.05 * 2 < p < 0.10 * 2$

or $0.10 < p < 0.20$

One-sample *z*-test

If n is very large (>200) or if σ is known, we can substitute a *z*-test for the *t*-test.

$$z = \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}} \sim N(0,1)$$

For a two-tailed test, reject H_o if

$z > z_{1-\alpha/2}$ **or**

$z < z_{\alpha/2}$

Example: Suppose we are interested in the mean age of a certain population. Suppose, we know that $\sigma^2 = 20$ and that the population is approximately normally distributed. Can we conclude that the mean age is different from 30?

H_o: $\mu = 30$
H_1: $\mu \neq 30$

Let $\alpha = 0.05$

Reject H_o if $z > 1.96$ or if $z < -1.96$.

Suppose we sample 10 individuals and get $\overline{X} = 27$.

$$\begin{aligned} z &\sim \frac{\overline{X} - \mu_o}{\sigma / \sqrt{n}} \\ &= \frac{27 - 30}{\sqrt{20} / \sqrt{10}} \\ &= \frac{-3}{1.4142} \\ &= -2.12 \end{aligned}$$

Decision: Reject H_o

Conclusion: We have evidence that the mean age is not equal to 30 years.

p-value:

a. **If $z \leq 0$, p = $2\Phi(z)$**

b. **If $z > 0$, p = $2[1 - \Phi(z)]$**