



**EARs and non-annual CFs: go down to periodic rate**

- If your EAR is 10% and your CFs are....
  - Semi-annual:
    - Semi-annual rate =  $1.10^{1/2} - 1 = 0.048809$  or 4.8809%
  - Quarterly:
    - Quarterly rate =  $1.10^{1/4} - 1 = 0.024114$  or 2.4114%
  - Monthly:
    - Monthly rate =  $1.10^{1/12} - 1 = 0.007974$  or 0.7974%
  - Daily:
    - Daily rate =  $1.10^{1/365} - 1 = 0.000261$  or 0.0261%

**Decisions, Decisions**

- You are looking at two savings accounts. One pays 5.25%, with daily compounding. The other pays 5.3% with semiannual compounding. Which account should you use?
  - First account:
    - EAR =  $(1 + .0525/365)^{365} - 1 = 5.39\%$
  - Second account:
    - EAR =  $(1 + .053/2)^2 - 1 = 5.37\%$
- Choose the first account, even though it has the lower quoted rate.

CF  
quarter  
Rate  
annual (EAR)  
12 month down to 9 month → 1/4 fractional

1 month  
1000 → 1200 20%  
APR = 20% × 12 = 240%  
EAR =  $[1.20^{12} - 1] × 100 = 791.61\%$

**Loan Shark**

- You can borrow \$500 from Johnny Roast Beef in exchange for paying \$650 next week. What is the APR and EAR of this transaction?

$$EAR = \left[ \left( \frac{650}{500} \right)^{52} - 1 \right] × 100 = 84,149,938.68\%$$

$$APR = \left[ \left( \frac{650}{500} \right)^{52} - 1 \right] × 100 = 1,560\%$$

↑ makes # of % age

**More on APR/EAR**

- The concept of converting APRs and EARs into the rate needed to solve a problem trips up many students.
- To see a comprehensive list of calculations, see "EAR v. APR" on Carmen.

**Mortgage Loans: 3 Basic Types**

- Interest-Only: Interest is paid every period; principal is paid at expiration (if at all). Frequently used during the housing bubble. Most bonds are examples of interest-only loans, also.
 

all principle paid at end
- Amortized: Each payment contains some principal and some interest. When you make the last payment, your balance hits zero and you own the asset in full. Car loans, mortgage loans, student loans, etc.
 

pay on amortization schedule
- Partially Amortized: Also known as a "balloon" mortgage. A "7/23 balloon" would be a 30-year mortgage where you make monthly payments for 7 years, then pay off the principal balance of the remaining 23 years, usually by refinancing.
 

paid off before end of period

**Amortized Loans: Fixed PMT** APR = 9% w/ monthly compounding

- If the payment is fixed, then the loan is like an annuity: see the "Finding the Payment" slide discussed earlier.
- The \$207.58 paid each month will be part principal and part interest
- N=60; I/Y=.75; PV=10,000; PMT=-207.58; FV=0
- The amortization schedule looks like this:

Period	Beginning Principal	Payment	Principal	Interest	Ending Principal
1	10,000.00	(207.58)	(132.58)	(75.00)	\$9,867.42
2	\$9,867.42	(207.58)	(133.58)	(74.01)	\$9,733.84
3	\$9,733.84	(207.58)	(134.58)	(73.00)	\$9,599.26
...	...	...	...	...	...
58	611.53	(207.58)	(100.58)	(107.00)	\$500.95
59	\$416.95	(207.58)	(100.58)	(107.00)	\$409.37
60	\$208.04	(207.58)	(100.58)	(107.00)	\$0.00

every month pay off loan little by little  
pay int./principle  
fixed payment  
very month pay off

→ same amt every month  
principle/interest amt always add up to pmt but amt in each changes  
-int. amt ↓ but principle amt ↑  
paying off principle in bigger amts

EAR = ~~nom.~~ 6.35%  
 0.5144% = month =  $[(1.0635)^{\frac{1}{12}} - 1] \times 100\%$

### Amortized Loans: Fixed PMT (cont'd)

- How about a mortgage loan? Assume a 30-year fixed mortgage at 6% for \$300,000.
- N=360; I/Y=0.5; PV=300,000; FV=0  
 CPT PMT = -1798.65

Period	Beginning Principal	Payment	Ending Principal	Interest	Ending Payment
1	\$ 300,000.00	(\$1,798.65)	(\$215.65)	(\$1,582.99)	\$298,784.35
2	\$298,784.35	(\$1,798.65)	(\$400.14)	(\$1,498.51)	\$298,404.20
3	\$298,404.20	(\$1,798.65)	(\$681.65)	(\$1,416.99)	\$298,009.55
...	...	...	...	...	...
358	\$ 3,342.44	(\$1,798.65)	(\$1,771.94)	(\$20.71)	\$1,570.50
359	\$1,570.50	(\$1,798.65)	(\$1,798.00)	(\$1.39)	\$1,789.20
360	\$1,789.20	(\$1,798.65)	(\$1,789.20)	(\$0.99)	\$0.00

Note how much smaller the principal payments are (as a fraction of the total payment) in the early periods.

underwater owe more than house is worth

### Partially Amortized Loans: Example

- You borrow \$400,000 on a house with a 7/23 balloon mortgage at 5.5% APR with monthly compounding.
- What is your monthly payment? It will be based on a 30-year mortgage: N=360, I/Y=0.458333, PV=400,000, FV=0
- CPT PMT = -2,271.156
- What is the balloon payment? After 7 years, there are 276 payments left (360-84).
- N=276, CPT PV = 355,263.54.



As int. rate ↓ home owners ~~can~~ refinance

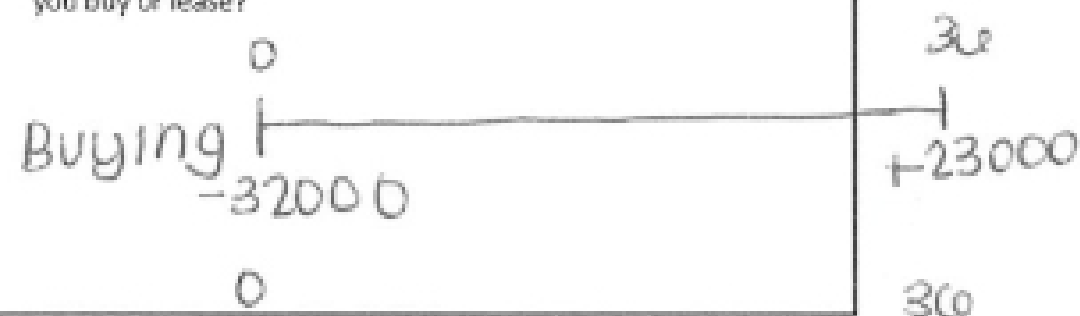
### Calculating Loan Payments

- You need a 30-year, fixed-rate mortgage to buy a new home for \$240,000. Your mortgage bank will lend you the money at a 6.35% APR. (APR is Annual Percentage Rate. To get the monthly rate, divide 6.35% by 12.) However, you can afford monthly payments of only \$1,150, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment.
- What would be the monthly payment needed to pay off this mortgage at the end of 30 years?
- What would be the payment if this were an interest-only loan?
- If we only pay \$1,150/month, what will the balloon payment be? Can you explain what happened?

$FV=0 \rightarrow pmt = 1493.37$   
 $240000 \times \frac{.0635}{12} \quad pmt = 1270$   
 or  $n = 360 \quad r = \frac{.0635}{12} \quad PV = 240000 \quad FV = -240000$   
 $pmt = 1270$   
 •  $pmt = 1150 \quad FV = 368936$   
 getting bigger so negative amortizing  
 $pmt = -1150 \quad r = .5291667 \quad PV = 240000$   
 $n = 360 \quad FV = 368936$

### Buy Vs. Lease

- After deciding to buy a new car, you can either lease the car or purchase it on a 3-year loan. The car costs \$32,000. The lease offer is for \$450/month for 36 months, with only \$99 due up front. If you buy the car with a 3-year loan, the APR is 7%. You believe that in 3 years the car will be worth \$23,000. Should you buy or lease?



lease  $-99 \quad -450/mo. \quad n=36 \quad r = \frac{7}{12} \quad pmt = 450 \quad FV=0$   
 $CPT PV = 14573.91 + 99 = 14672.91$   
 $PV(\text{Buying}) = -32000 + \frac{23000}{(1 + \frac{.07}{12})^{36}}$   
 $n=36 \quad r = \frac{7}{12} \quad pmt = 0 \quad FV = 23000$   
 $PV = 18654.31 - 32000 = -13345.69$

leasing costs more here