

**CHAPTER 4:  
TIME VALUE OF MONEY:  
VALUING CASH FLOW STREAMS**

Business Finance 3220

**3 types of multiple cash flow streams**

- **Annuity:** cash flows are all the same amount, and spread out equally over a finite period of time. \$1000 per year for 5 years, e.g. *regular intervals w/ end point*
- We can handle annuities using the Time Value of Money keys on the BA-II Plus, specifically the PMT key. *frequency/dollar amt same w/ end*
- **Perpetuity:** cash flows are same amount, over regular intervals, but forever. *ex. stocks*
- **Multiple, uneven cash flows:** use the cash flow (CF) function. *streams of annuities w/ unequal amts of cash flows over unequal amts of time*

*ex. student loans, car loan, mortgage*

**Annuities and Perpetuities: Formulas**

- Perpetuity:  $PV = C / r$
- Growing perpetuity:  $PV = C / (r - g)$  *cash flow grow at constant rate each year*
- Ordinary Annuities:  

$$PV = C \times \frac{1}{r} \left[ 1 - \frac{1}{(1+r)^N} \right] \quad FV = C \times \frac{1}{r} [(1+r)^N - 1]$$
- Growing Annuities:  

$$PV = C \times \frac{1}{r-g} \left[ 1 - \left( \frac{1+g}{1+r} \right)^N \right]$$

$$FV = C \left[ \frac{(1+r)^N - (1+g)^N}{r-g} \right]$$
- Where C is the cash flow, r is the interest rate and N is the number of periods

*growth rate*

**Annuities and the Financial Calculator**

- You can use the PMT key on the calculator for the annuity payment (C, in the formula)
- The sign convention still holds
- \*If the calculation is a growing annuity, you must solve using the formula (unfortunately). \*

*deposit 1000*

*APR = annual % rate = % rate x #month/year*

**Calculating Annuity Values**

- If you deposit \$10,000 at the end of each of the next 10 years into an account paying 10% interest, how much money will you have in the account in 10 years?
- What if the deposits grew at 5% per year?

*FV =*  $C \times \frac{1}{r} [(1+r)^N - 1] = 10000 \times \frac{1}{.10} [(1+.10)^{10} - 1]$   
 $= 159374.20$

*FV =*  $C \left[ \frac{(1+r)^N - (1+g)^N}{r-g} \right] = 10000 \left[ \frac{(1.10)^{10} - (1.05)^{10}}{0.10 - 0.05} \right]$   
 $= 192969.57$

**Finding the Payment**

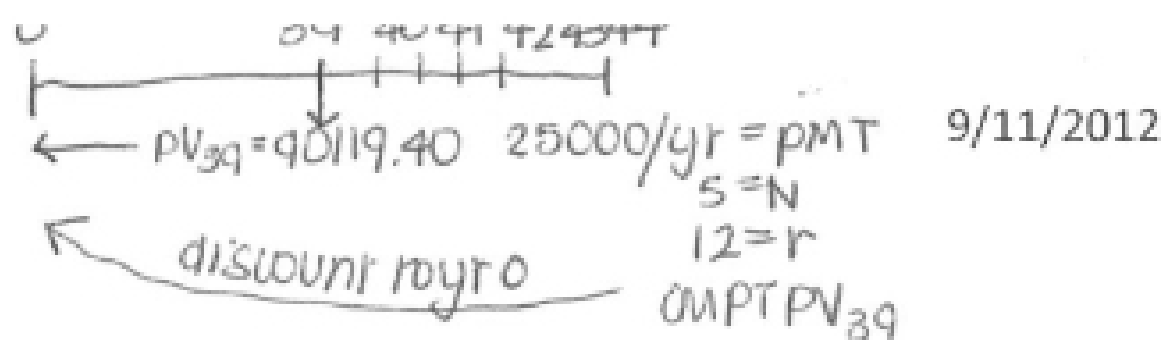
*APR =  $\frac{2}{3}\% \times 12 = 8\%$*

- Suppose you want to borrow \$20,000 for a new car. You can borrow at a rate of 2/3% per month. If you take a 4 year loan, what is your monthly payment? *4x12 = every month for 48 month*
- $20,000 = C [1 - 1 / 1.006667^{48}] / .006667$
- $C = 488.26$
- $4(12) = 48 N; 20,000 PV; .66667 I/Y; CPT PMT = -488.26$

$$PV_0 = \frac{7011.71}{(1+0.12)^{39}} = 1084.71$$

$$CF_0 = 0 \quad CF_1 = 0 \quad CF_2 = 25000 \quad I = 12$$

$$F_1 = 39 \quad F_2 = 5$$



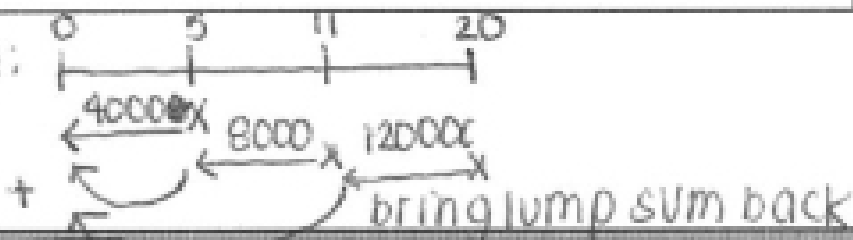
### Multiple Uneven Cash Flows – Using the Calculator

- Another way to use the financial calculator for uneven cash flows is to use the cash flow keys
- Press CF and enter the cash flows beginning with year 0 (CF<sub>0</sub>).
- You have to press the "Enter" key for each cash flow
- Use the down arrow key to move to the next cash flow
- The "F" is the number of times a given cash flow occurs in consecutive periods
- Use the NPV key to compute the present value by entering the interest rate for I, pressing the down arrow and then compute
- When done, clear the cash flow value by pressing <2nd> <CE|C> (CLR WORK)

### Multiple Cash Flows – PV

- You are considering an investment that will pay you \$1000 in one year, \$2000 in two years and \$3000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?
- Leave CF<sub>0</sub> blank. This is the value you want to solve for when you compute NPV (net present value) below.
- CF<sub>0</sub>=1000 <ENTER> <arrow down>
- F<sub>01</sub>=1. Since this is the default value, just <arrow down>
- CF<sub>02</sub>=2000 <ENTER> <arrow down>; F<sub>02</sub>=1 <arrow down>
- CF<sub>03</sub>=3000 <ENTER> <arrow down>; F<sub>03</sub>=1 <arrow down>
- Hit <NPV>. Enter 10 for I and hit <ENTER>
- <arrow down> and hit <CPT>
- NPV=4815.927

example:



### Saving For Retirement

- You are offered the opportunity to put some money away for retirement. You will receive five annual payments of \$25,000 beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?
- Note: the first cash flow is in year 40. This means you get zero for 39 years.
- Use the cash flow keys:
- CF<sub>0</sub>=0; CF<sub>1</sub>=0; CF<sub>2</sub>=0; F<sub>01</sub>=39; CF<sub>2</sub>=25000; F<sub>02</sub>=5; NPV; I=12; CPT
- NPV = 1084.71
- Remember to hit <ENTER>, then arrow down after each entry.

as of year 5  $CF_0 = 0 \quad CF_1 = 0 \quad CF_2 = 7000$

$F_01 = 6 \quad F_02 = 10$

$PV = 39889.72$

### Calculating PV with the CF Function

- A 5-year annuity of ten \$7,000 semiannual payments will begin 8 years from now, with the first payment coming 8.5 years from now. If the discount rate is 5.105% every six months, what is the value of the annuity 5 years from now? What is the value 3 years from now? What is the value today?

ordinary annuity - annuity cash flow occur at end of period

5 yr. annuity of 10 semiannual payments \$7000 payments 8 yr from now first pay at 8.5



$CF_0 = 0 \quad CF_1 = 0 \quad CF_2 = 7000$

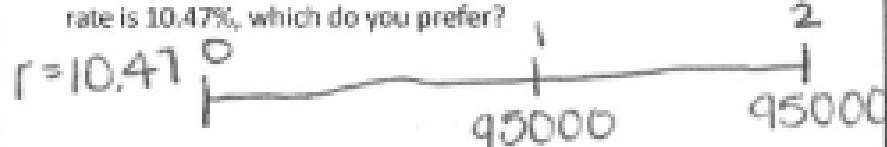
$F_01 = 6 \quad F_02 = 10$

$I = 5.105$

value today = 24245.28

### Comparing Cash Flow Streams

- Your new firm is offering you two different salary arrangements. You can have \$95,000/year for the next two years, or \$70,000/year for the next two years along with a \$45,000 signing bonus today. The bonus is paid immediately, and the salary is paid at the end of each year. If your discount rate is 10.47%, which do you prefer?



$n = 2 \quad pmt = 95000 \quad FV = 0 \quad r = 10.47$

$CMT \quad PV = 102841.95$

$pmt = 70000 \quad n = 2 \quad FV = 0 \quad r = 10.47$

$PV = 120725.64 + 45000$

$= 165725.64$

### Chapter 4, Problem 34

- You have just turned 22 years old, have just received your bachelor's degree, and have accepted your first job. Now you must decide how much money to put into your retirement plan. Every dollar in the plan earns 7%/year. You cannot make withdrawals until age 65. You plan to live to be 100, and estimate that you'll need \$100,000/year in retirement. If you contribute the same amount each year, how much will you need to contribute each year to fund your retirement?

- Two parts:
- 1. Find the PV of what you need at age 65 to fund your 35-year retirement.
- 2. Change that number to the PV and calculate the PMT you need each year from age 22-65 (43 years) to get there.

$PV = 0 \quad n = 43 \quad r = 7\%$

$FV = 1294767.23 \quad pmt = 5225.54$

$PV = 1294767.23$

$w \quad pmt = 100000/yr \quad n = 35 \quad r = 7\% \quad FV = 0$