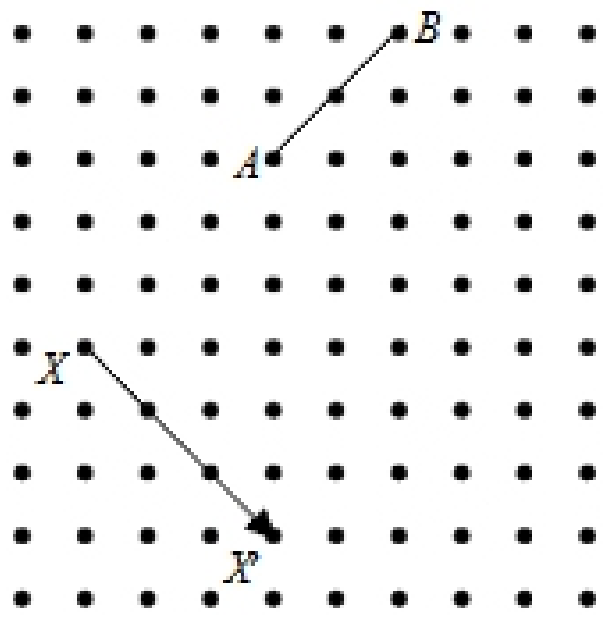
Section 12.1 – Translations and Rotations

Any rigid motion that preserves length or distance is an *isometry* (meaning "equal measure"). In this section, we will investigate two types of isometries: *translations* and *rotations*.

Translations

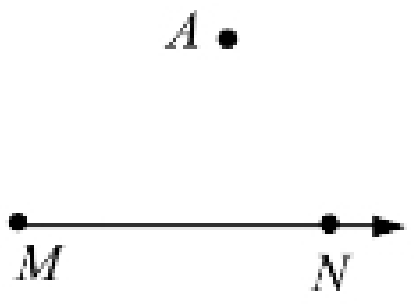
- A *translation* is a motion of a plane that moves every point of the plane a specified distance in a specified direction along a straight line.
- Properties of Translations
  - A figure and its image are congruent.
  - The image of a line is a line parallel to it.

Example: Find the image of $\overline{AB}$ under the translation from X to X' pictured on the dot paper below.



Constructions of Translations

Example: Construct the image A' of point A in the direction and magnitude of vector $\overrightarrow{MN}$.

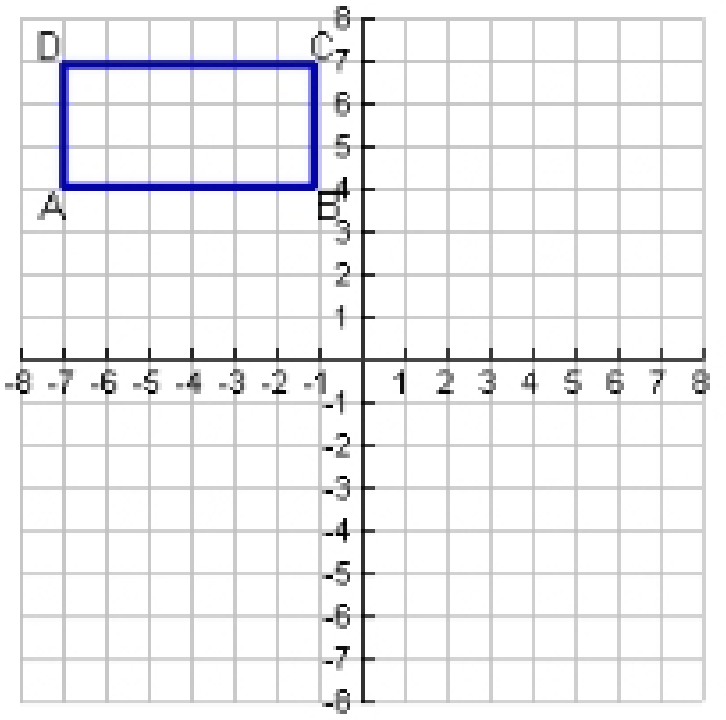


Coordinate Representation of Translations
You can use formulas when the translation occurs in the rectangular coordinate system.

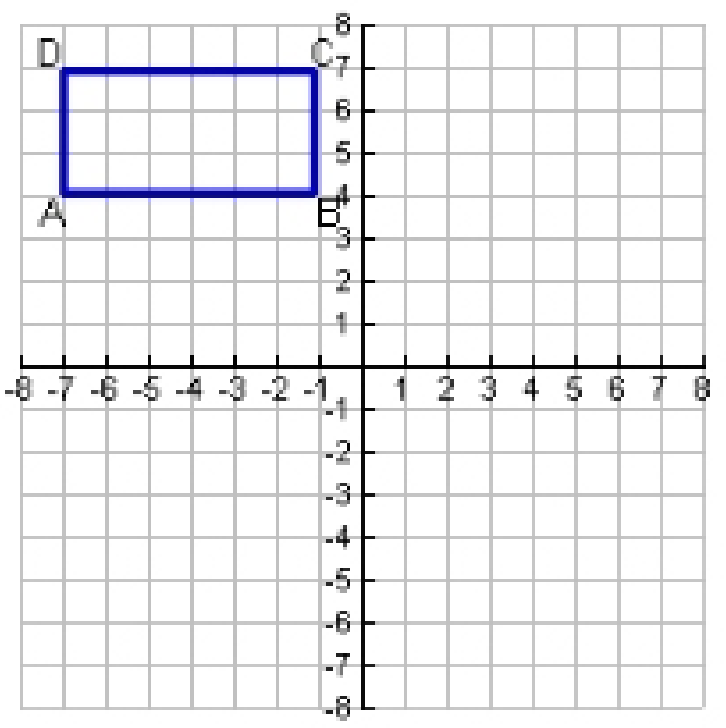
Property of a Translation in a Coordinate System
A translation is a function from the plane to the plane such that to every point (x, y) corresponds the point $(x + a, y + b)$, where a and b are real numbers.

Example: Find the coordinates of the image of the vertices of quadrilateral $ABCD$ in the figure below under the translations given. Draw the image in each case.

- $(x, y) \rightarrow (x + 2, y - 3)$



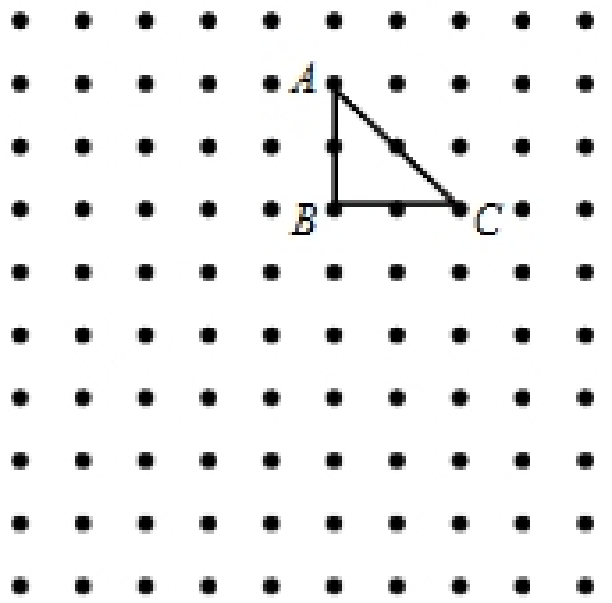
- a translation determined by the slide arrow from $A(1, 2)$ to $A'(3, -1)$



Rotations

- A *rotation* is a transformation of the plane determined by rotating the plane about a fixed point, the *center*, by a certain amount in a certain direction.
- Usually, a positive measure is a counterclockwise turn and a negative measure is a clockwise turn.
- A rotation of 360° about a point will move any point (and figure) onto itself. Such a transformation is an *identity transformation*. A rotation of 180° is a *half-turn*.

Example: Find the image of $\triangle ABC$ under the rotation with center O.



Construction of a Rotation

Example: Construct the image of point P under a rotation with center O through the angle and in the direction given in the figure below.