MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.01—Introduction to EECS I
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Lecture 7 Notes

Constraint Systems and Circuits

Circuits

Electrical circuits are made up of components, such as resistors, capacitors, inductors, and transistors, connected together by wires. You can make arbitrarily amazing, complicated devices by hooking these things up in different ways, but in order to help with analysis and design of circuits, we need a systematic way of understanding how they work.

As usual, we can't comprehend the whole thing at once: it's too hard to analyze the system at the level of individual components, so, again, we're going to build a model in terms of primitives, means of combination, and means of abstraction. The primitives will be the basic components, such as resistors and op-amps; the means of combination is wiring the primitives together into circuits. We'll find that abstraction in circuits is a bit harder than in software or linear systems: separately designed parts of a circuit tend to influence one another when they are connected together, unless you design very carefully. We'll explore a number of examples of when and how the abstractions can help us, but also when they can leave out important detail and require different models.

Constraint Models

So far, we have looked at a number of different models of systems. We have thought of software procedures as computing functions, of a robot "brain" as performing a transduction from a stream of inputs to a stream of outputs, and of linear systems as a special subclass of transductions that we can analyze for stability and other properties. In each case, we were able to construct or analyze the behavior of sub-parts of the system, as functions or transductions, and then abstract away from their implementations, use them to build more complex systems, and use the understanding of the components to understand the larger system.

Now we're going to consider a different class of systems that has a kind of modularity, but where, typically, you have to have a description of the entire system in order to say what is going to happen in a local piece of it. We will be able to view the sub-parts as putting "constraints" on the overall global behavior of the system; once enough pieces are put together and their constraints are taken together, the behavior of the entire system will be specified.

One intuitive example is a set of rigid rods connected together with pins, all resting flat on a table. If we specify the x, y coordinates of the end points of one rod, and the lengths of the other rods, and the way in which they're connected together, we have described a set of constraints on the positions of all the rods. If, for example, we connect 4 rods of length 1 in a square, then the positions of the other rods are not completely specified, because the square can be squashed into a number of different rhombuses. On the other hand, if we connect only three rods into a triangle, then the position of the third vertex will be completely specified.

We will use this way of thinking about and specifying the behavior of a system to understand simple electrical circuits as systems of constraints.

Voltage and current

Voltage is a difference in electrical potential between two different points in a circuit. We will, generally speaking, pick some point in a circuit and say that it is "ground" or has voltage 0. Now, every other point has a voltage defined with respect to ground. Because voltage is a relative concept, we could pick *any* point in the circuit and call it ground, and we would still get the same results.

Current is a flow of electrical charge through path in the circuit. A positive current in a direction is generated by negative charges (electrons) moving in the opposite direction.[1] We're not going to worry about the details of what particles are doing what (until we get to semiconductors, in another class). We'll just have to be careful when we draw and describe circuits to label the directions of the currents we're talking about.

Static circuit model

A circuit is made up of a set of components, wired together in some structure. Each component has a current flowing through it, and a voltage difference across its two terminals (points at which it is connected into the circuit). Each type of component has some special characteristics that govern the relationship between its voltage and current.

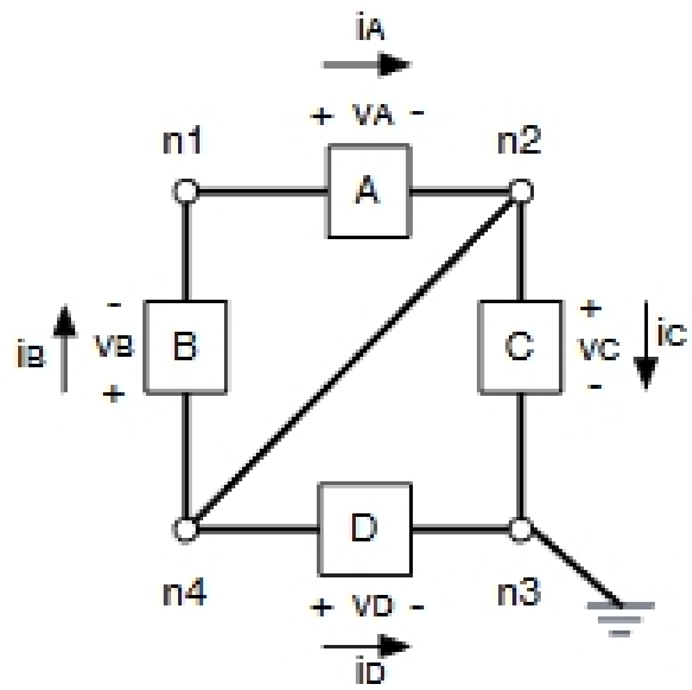
In general, circuits have dynamic behavior. That is, the voltages and currents in the system change over time. Models for the dynamic behavior of circuits are usually in the form of difference or differential equations. For now, we will consider a simpler case, one where we have assumed that the dynamic behavior has settled to an equilibrium state.

In this equilibrium setting, we will consider the case where combinations components, and the way they are connected, provides a set of constraints on the equilibrium state of the circuit. We'll work through this view by starting with the constraints that come from the structure, and then examining constraints for two simple types of components.

Conservation laws

One set of constraints in circuit problems stems from enforcing a *conservation law* on the circuit currents, often referred to as *Kirchoff's Current Law (KCL)*. This conservation law holds true, no matter what kinds of components we use in our circuit. We'll describe the conservation law using the circuit in figure 1A. For now, don't worry about what's in the components labeled A through D. You can see that we've labeled the current through each component with an arrow, and named it i_x. We can choose these arrows to point in any direction we like, as long as we treat them consistently (we'll say this more precisely later). For each component, we can also talk about the *voltage drop* across the component, which we've labeled v_x. It is the potential difference between

[1] At the semi-conductor level, it can also be viewed in an oversimplified way as as "holes" or positive charges moving in the direction of the current.



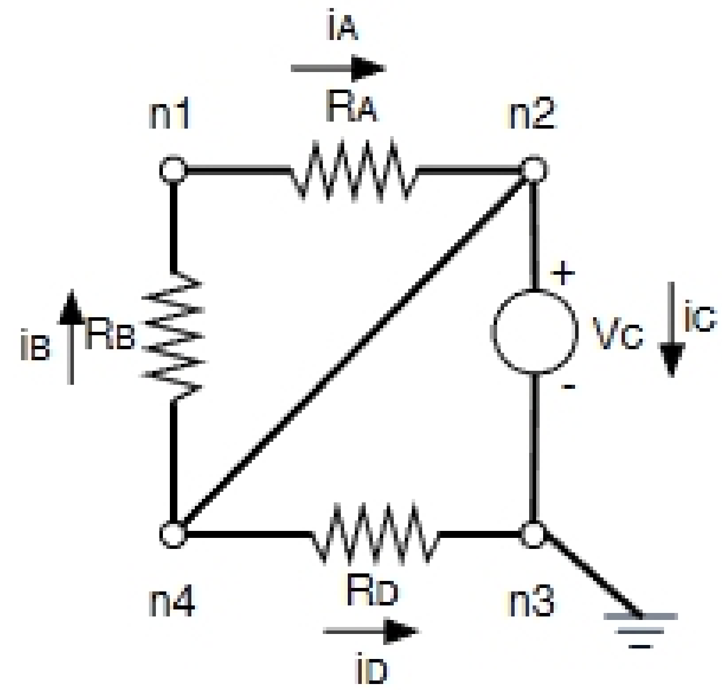


Figure 1: A. Circuit with four components. B. Circuit with three resistors and a voltage source.

the terminal labeled '+' and the terminal labeled '-', which should agree with the direction of the current arrow for the component, flowing from '+' to '-'.

Kirchhoff's Current Law (KCL)

Each place in a circuit where two or more components connect is called a *node*, and we can label each of them with a node name (like n_1, n_2, etc.).

Kirchhoff's current law is a source of constraints that govern the behavior of a circuit. It says that:

> The algebraic sum of current entering any node must be zero.

We can write a KCL equation for each node in our circuit. Since there is a wire connecting nodes n_2 and n_4, in fact they have the same voltage, and can be considered as a single node for the purposes of analysis. So, we have, at node n_1, that $i_A - i_B = 0$. At node n_2, because it's the same as node n_4, things are a little tricky. We have incoming current from A, and current flowing out through B, C, and D. So, we get the equation: $i_B + i_C + i_D - i_A = 0$. Remember that the signs of these currents and their directions are all a matter of convention: we don't actually know yet whether the voltage at n_2 will be higher than the voltage at n_1 or not.

Node n_3 is connected to the *ground* symbol (that stack of three horizontal lines), which means we will treat it as having voltage 0. So, we can speak, now, of the voltage at node n_1, which we'll write v_1, which is really the voltage difference between n_1 and n_3. We will say that we've *solved* a circuit, when we've been able to figure out *the voltages at all the nodes and the currents through all the components.*

Elements

Now we need to know what the actual elements of the circuit are, in order to know how it is going to behave. In this course, we'll start by considering two very basic elements: independent voltage