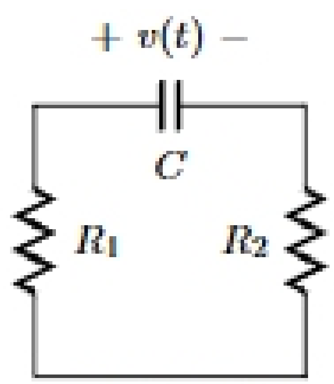
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2006

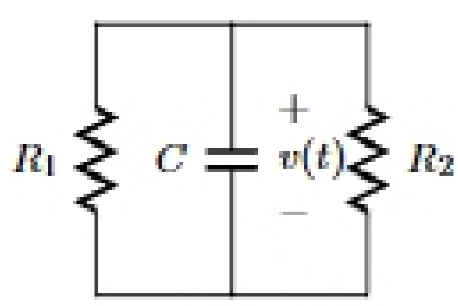
Problem Set #7

Issued 3/22/06 – Due 4/5/06

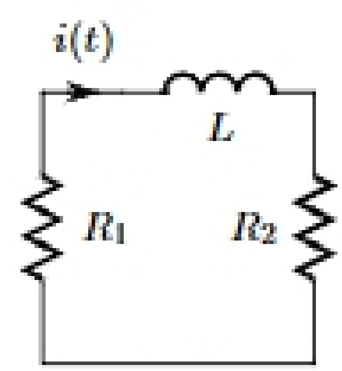
Exercise 7.1: Each network shown below has a non-zero initial state at $t = 0$, as indicated. Find the network states for $t \geq 0$. Hint: what equivalent resistance is in parallel with each capacitor or inductor, and what decay time results from this combination?



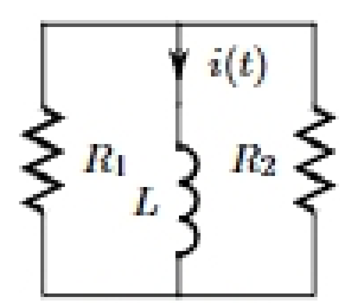
$v(0) = V$



$v(0) = V$

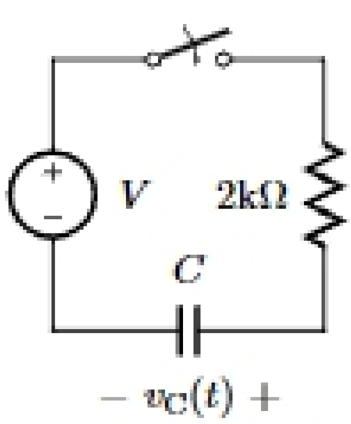


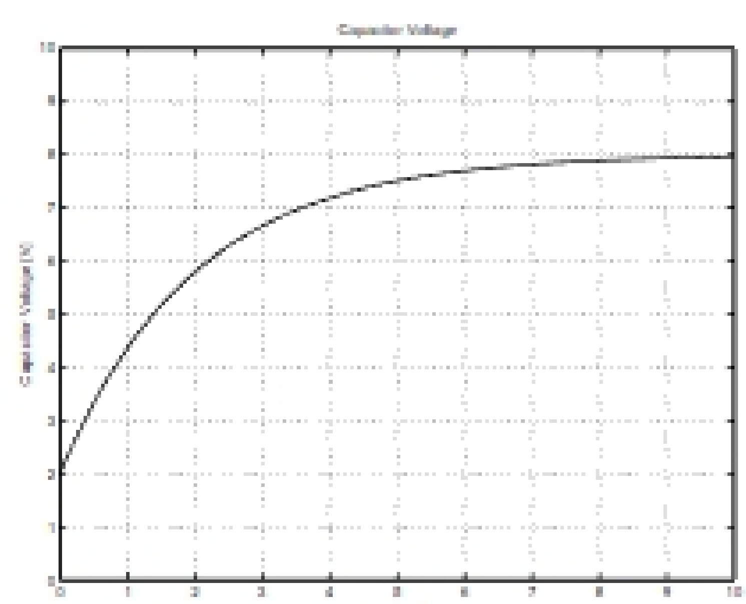
$i(0) = I$



$i(0) = I$

Exercise 7.2: The network shown below contains a voltage source having amplitude V, an ideal switch, a 2-kΩ resistor and a capacitor having capacitance C, all in series. At $t = 0$ the switch closes, after which the capacitor voltage v_C is measured as shown below. From the measured voltage, determine V, C, and the capacitor voltage before the switch closed. *Note: the last page of this problem set contains a larger graph of the capacitor voltage. It can be turned in with your problem set solutions.*





Problem 7.1: This problem examines the relation between transient responses of linear systems. The network shown below is first driven by a current step at $t = 0$, then driven by a current ramp at $t = 0$, and finally driven by the current step plus the current ramp at $t = 0$. In the first two cases, the inductor has zero initial current, as indicated.

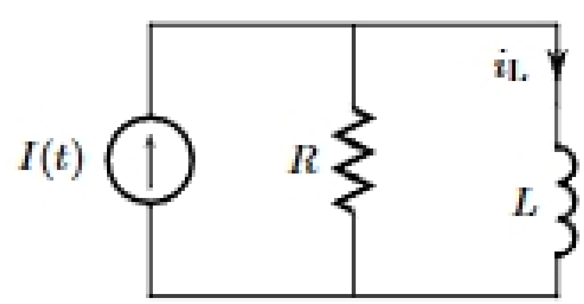
(A) Find the inductor current $i(t)$ for $t \geq 0$ in response to the current step $I(t) = I_\circ u_{-1}(t)$. Assume that $i(0) = 0$.

(B) Find the inductor current $i(t)$ for $t \geq 0$ in response to the current ramp $I(t) = I_\circ \alpha t u_{-1}(t)$. Again assume that $i(0) = 0$.

(C) The step input can be constructed from the ramp input according to $I_{\text{Step}}(t) = \frac{1}{\alpha}\frac{d}{dt}I_{\text{Ramp}}(t)$. Show that their respective responses are related in a similar manner.

(D) Would the result from Part C hold if $i(0) \neq 0$? Why or why not?

(E) Finally, find the inductor current $i(t)$ for $t \geq 0$ in response to the current step plus the current ramp, that is, in response to $I(t) = I_\circ(1 + \alpha t)$ for $t \geq 0$. This time assume that $i(0) = i_\circ$. Hint: think superposition.



Problem 7.2: The circuit shown below can be used to regulate the current through an inductor. (The switches model transistors.) Typical applications include the regulation of currents in motors, solenoids and loud speakers, all of which have inductive windings. We will analyze the circuit assuming that it operates in a cyclic manner with switching period T. During the first part of each period, which lasts for a duration DT, switches S1 and S4 are on while switches S2 and S3 are off. During the second part of each switching period, which lasts for a duration $(1-D)T$, switches S1 and S4 are off while switches S2 and S3 are on. Note that $0 \leq D \leq 1$.

(A) Assume that D is constant and that the circuit has been operating long enough to reach a cyclic steady state by $t = 0$, at which point a new switching period begins. In terms of the unknown $i(0)$, determine $i(t)$ for $0 \leq t \leq T$.

(B) Use your result from Part (A), and the fact that the circuit operates in a cyclic steady state to determine $i(0)$. Note that with this result, and that from Part (A), $i(t)$ is completely determined.

(C) Find the average value of $i(t)$ over the period $0 \leq t \leq T$. Hint: is it necessary to average the result from Part A, or is there a faster method to find the average?

(D) Suppose that the circuit has been operating with $D \equiv D_1$ for a time long enough to reach a cyclic steady state by $t = 0$. Suppose that D switches to $D = D_2$ at $t = 0$, just as a new switching period begins. In this case, determine $i(t)$ for $t \geq 0$. Hint: can you use your result from Parts (A) and (B) as a particular solution over the interval $0 \leq t$?