5. For the series RL network

a) Let $v_s(t) = V_m \cos(\omega t + \phi)$

$$= Re\{\underline{V}\, e^{j\omega t}\}$$

where $\underline{V} = V_m e^{j\phi}$

b) Then the steady state current has the form:

$$i(t) = I_m \cos(\omega t + \beta\}$$

$$= Re\{\underline{I}\, e^{j\omega t}\}$$

where $\boxed{\underline{I} = I_m e^{j\beta}}$

c) The circuit equation was:

$$L \frac{d\,i(t)}{dt} + R\,i(t) = V_m \cos(\omega t + \phi)$$

d) Substituting for $v(t)$ and $i(t)$ gives:

$$L \frac{d}{dt}\left[Re\{\underline{I}\, e^{j\omega t}\}\right] + R\left[Re\{\underline{I}\, e^{j\omega t}\}\right]$$

$$= Re\{\underline{V}\, e^{j\omega t}\}$$

or

$$\mathrm{Re}\left\{\frac{d}{dt}\left[L\underline{I}e^{j\omega t}\right]\right\} + \mathrm{Re}\left\{R\underline{I}e^{j\omega t}\right\}$$
$$= \mathrm{Re}\left\{\underline{V}e^{j\omega t}\right\}$$

$$\therefore \mathrm{Re}\left\{j\omega L\underline{I}e^{j\omega t}\right\} + \mathrm{Re}\left\{R\underline{I}e^{j\omega t}\right\}$$
$$= \mathrm{Re}\left\{\underline{V}e^{j\omega t}\right\}$$

$$\therefore \mathrm{Re}\left\{j\omega L\underline{I}e^{j\omega t} + R\underline{I}e^{j\omega t}\right\}$$
$$= \mathrm{Re}\left\{\underline{V}e^{j\omega t}\right\}$$

e) The arguments must be equal

$$\therefore j\omega L\underline{I}\cancel{e^{j\omega t}} + R\underline{I}\cancel{e^{j\omega t}} = \underline{V}\cancel{e^{j\omega t}}$$

or $$j\omega L\underline{I} + R\underline{I} = \underline{V}$$

$$\boxed{[R + j\omega L]\underline{I} = \underline{V}}$$

$$\therefore \quad \boxed{\underline{I} = \frac{V}{R + j\omega L}}$$

f) We can write

$$R + j\omega L = \sqrt{R^2 + \omega^2 L^2}\, e^{j\theta}$$

where $\theta = \tan^{-1} \frac{\omega L}{R}$

$$\therefore \quad \underline{I} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}\, e^{j\theta}}$$

$$= \frac{V}{\sqrt{R^2 + \omega^2 L^2}}\, e^{-j\theta}$$

$$\therefore \quad I_m e^{j\beta} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\, e^{j\phi} e^{-j\theta}$$

$$\boxed{= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\, e^{j(\phi - \theta)}}$$

or $$\boxed{\underbrace{I_m \cos(\omega t + \beta)} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)}$$

or $$\boxed{i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta) \qquad \theta = \tan^{-1} \frac{\omega L}{R}}$$