

1.
  - a. Assume R1 and R2 are union-compatible. Minimum is  $\max(N1, N2)$ , maximum is  $N1 + N2$ .
  - b. Assume R1 and R2 are union-compatible. Minimum is 0, maximum is  $\min(N1, N2)$ .
  - c. Assume R1 and R2 are union-compatible. Minimum is 0, maximum is  $N1$ .
  - d. No assumptions needed. Minimum and maximum are the same,  $N1 * N2$ .
  - e. Assume there's a field called "a". Minimum is 0, maximum is  $N1$ .
  - f. Assume there's a field called "a". Minimum is 1, maximum is  $N1$ .
  - g. Assume R1's fields are a superset of R2's. Minimum is 0, maximum is  $N1$ .
  
2. These are not necessarily the most efficient solutions! And because of the nature of the relational algebra, other solutions may work as well.
  - a.  $\pi_{\text{name}}(\pi_{\text{eid}}(\text{employee}) \bowtie \text{works} \bowtie \pi_{\text{cid}}(\sigma_{\text{name}=\text{'First Bank Corporation'}}(\text{company})))$
  - b.  $\pi_{\text{eid}}(\text{employee} \bowtie \text{works} \bowtie \sigma_{\text{city}=\text{'Wilmington'} \vee \text{city}=\text{'Philadelphia'}}(\text{company}))$
  - c.  $\pi_{\text{eid}}(\sigma_{\text{city}=\text{'Philadelphia'}}(\text{employee}) \bowtie \text{works} \bowtie \sigma_{\text{city}=\text{'Wilmington'}}(\text{company}))$
  - d.  $\pi_{\text{cid}}(\text{company} \bowtie \text{works} \bowtie \sigma_{\text{city}=\text{'Wilmington'} \vee \text{city}=\text{'Philadelphia'}}(\text{employee}))$
  - e. Set division can solve this:  
 $\pi_{\text{cid}}(\pi_{\text{cid, employee, city}}(\text{company} \bowtie \text{works} \bowtie \text{employee}) / \pi_{\text{city}}(\text{employee}))$
  - f.  $\pi_{\text{cid}}(\pi_{\text{cid, employee, city}}(\text{company} \bowtie \text{works} \bowtie \text{employee}) / \pi_{\text{city}}(\sigma_{\text{age} > 50}(\text{employee})))$
  - g. We don't actually need to do anything special:  
 $\pi_{\text{manager\_id, managee\_id}}(\text{manages})$
  - h. There are several different ways to do this. Here's one with a rename of a cross-product:  
 $\pi_{\text{eid1, eid2}}(\sigma_{\text{cid1}=\text{cid2} \wedge \text{eid1} \neq \text{eid2} \wedge \text{salary1} > \text{salary2}}(\text{workPairs}))$
  
3. In words:
  - a. The names of companies employing people that live in Wilmington and make more than \$100,000/year.
  - b. The names of companies employing people that live in Wilmington and make either more than \$100,000/year or less than \$10,000/year.
  - c. The names and salaries of employees with managers that live in Wilmington. (Note: there should have been a join to Works to recover salary. An acceptable answer is also "The names of employees with managers that live in Wilmington".)
  - d. Average salaries grouped by city.

- e. Pairs of employees such that one is making over \$1,000,000/year and the other is making \$10,000/year.