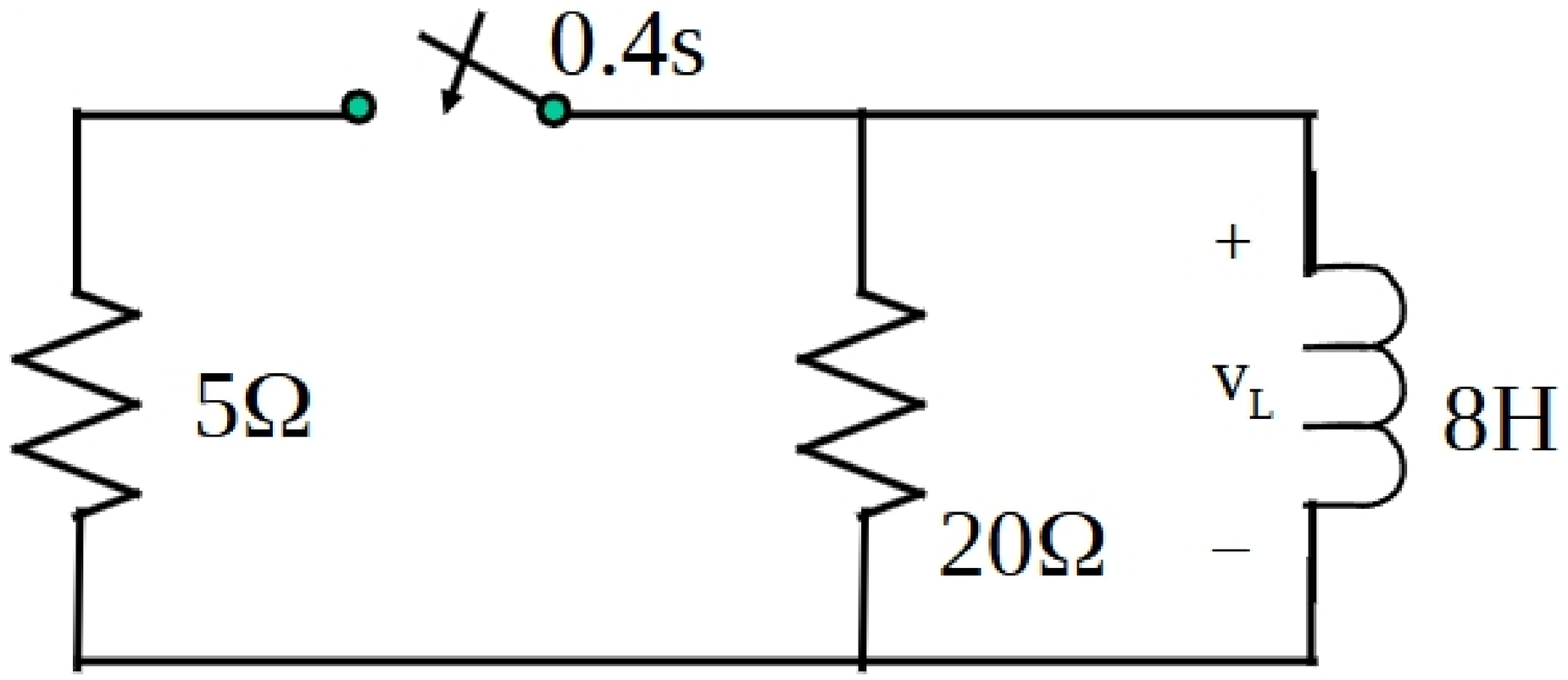
Source-free and step response

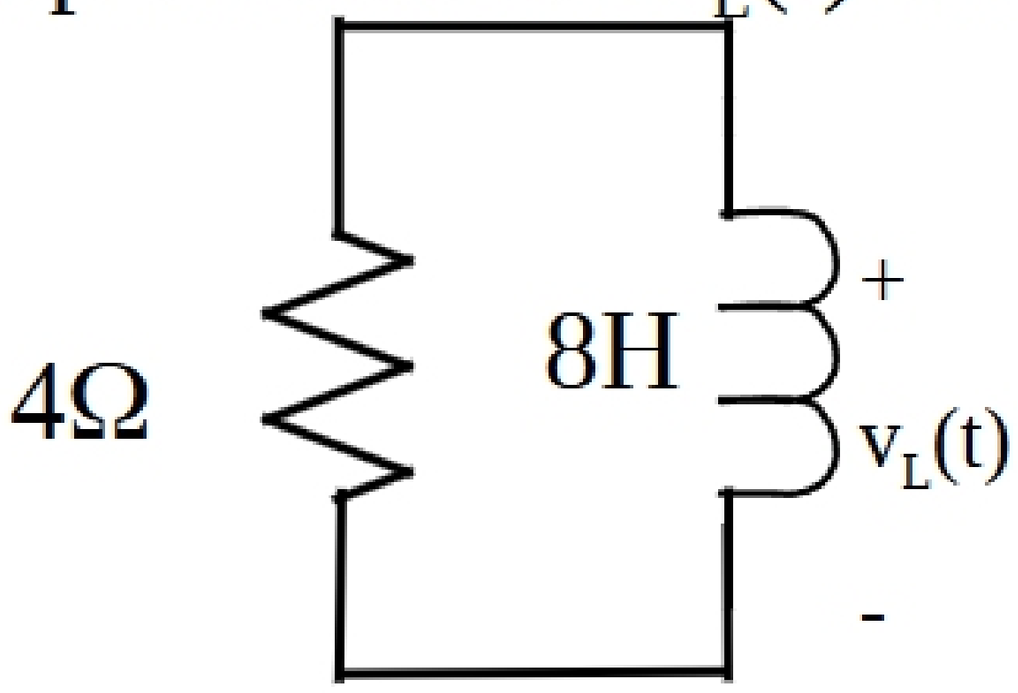
Example: Find $i_L(t)$ and $v_L(t)$ for $t \geq 0$ if $i_L(0^-) = 10A$, and the switch closes at t = 0.4s



Step1 : Compute R_{TH} for $t \geq 0.4s$

$$R_{TH} = 5\Omega \ // 20\ \Omega = 4\ \Omega$$

Step 2: Evaluate $i_L(t)$ for $t \geq 0.4s$



$$i_L(t) = [e^{-(R_{TH}/L)(t-t_0)}]\, i_L(t_0)$$

Where $t_0 = 0.4$ s. But $i_L(0.4)$ is unknown

Step 3: Evaluate $i_L(t)$ for $0 \leq t \leq 0.4$ s.

$$R_{TH} = 20\ \Omega$$

$$\tau = \frac{L}{R_{TH}}$$

$$i_L(t) = [e^{-(20/8)(t-0)}]\, i_L(0^+)$$

$$i_L(t) = 10\, e^{-2.5t}\ A$$

At $t = 0.4$ s, $i_L(0.4) = 10e^{-1} = 3.68$ A

Step 4: Put solution for $i_L(0.4)$ into equation for $i_L(t)$ $t \geq 0.4$ s.

$$i_L(t) = [e^{-(4/8)(t-t_0)}]\, i_L(0.4)$$

$$i_L(t) = 3.68\, e^{-0.5(t-0.4)}\ A$$

Step 5: Compute $v_L(t)$ from Ohm's Law

When $0 \le t \le 0.4$ s : $v_L(t) = -R_{TH}\, i_L(t)$

$v_L(t) = -(20)(10)[e^{-2.5t}] = -200\, e^{-2.5t}$ V

When $t \ge 0.4$ s $v_L(t) = -R_{TH}\, i_L(t)$

$$v_L(t) = -(4)3.68\, e^{(-0.5)(t-0.4)}$$

$$v_L(t) = -14.72\, e^{(-0.5)(t-0.4)}\ \text{V}$$

Unit step functions

$$u(t) = \begin{cases} 1 & t \ge 0 \\ 0 & t < 0 \end{cases} \qquad u(t - t_0) = \begin{cases} 1 & t \ge t_0 \\ 0 & t < t_0 \end{cases}$$

Examples: $v(t) = 10\, u(t)$ V

$$v(t) = \begin{cases} 10\text{V} & \text{if } t \ge 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$$i_L(t) = 3.68\, e^{-0.5(t-0.4)}\ \text{A}$$

$$i_L(t) = 3.68\, e^{-0.5(t-0.4)}\, u(t-0.4)\ \text{A}$$