SSS (cont.)

A sinusoidal function: $x(t) = A\cos(\omega t+\phi)$ can be represented as,

$$x(t) = Re[Ae^{j(\omega t+\phi)}] = Re[Ae^{j\omega t}\, e^{j\phi}]$$

For constant and known angular frequency, ω,

$$\text{Phasor of } x(t) = \underline{X} = \mathbf{X} = [Ae^{j\phi}] = A\angle\phi$$

Note: The $\mathbf{e^{j\omega t}}$ term of , $\mathbf{A_n e^{j\omega t} e^{j\phi}}$, is contained in all terms of the differential equation that describes the behavior of any first or second order circuit. Therefore, $\mathbf{e^{j\omega t}}$ can be cancelled from all terms, but it is understood the frequency remains unchanged.

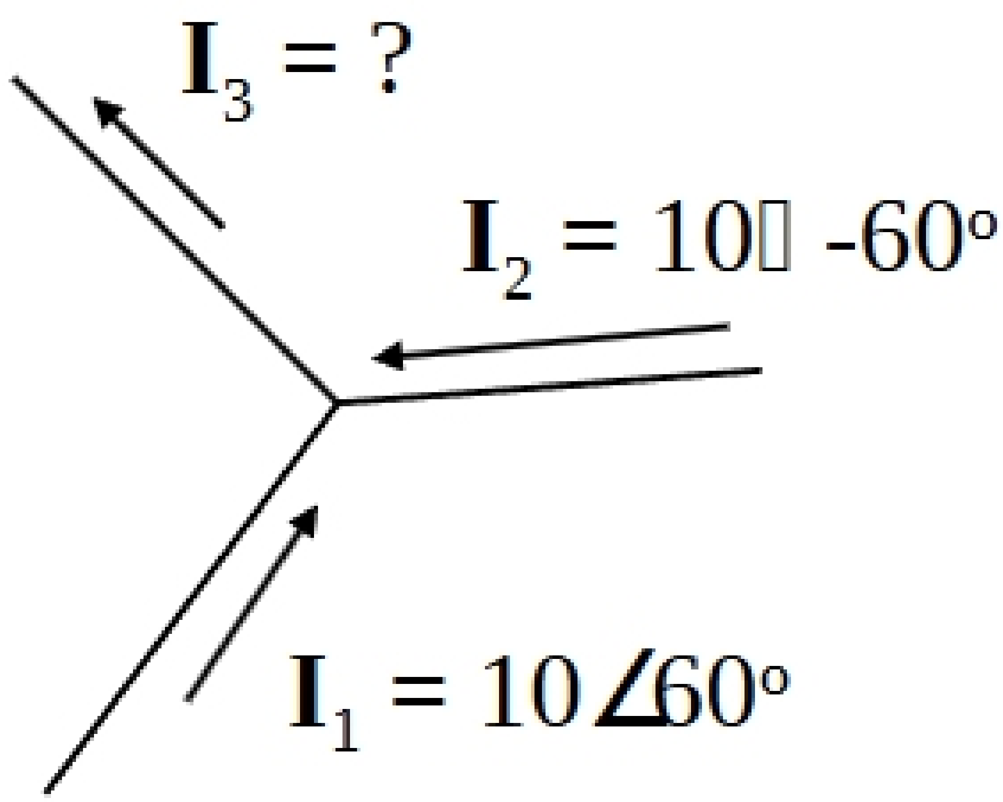
<u>Summation of Phasors</u>

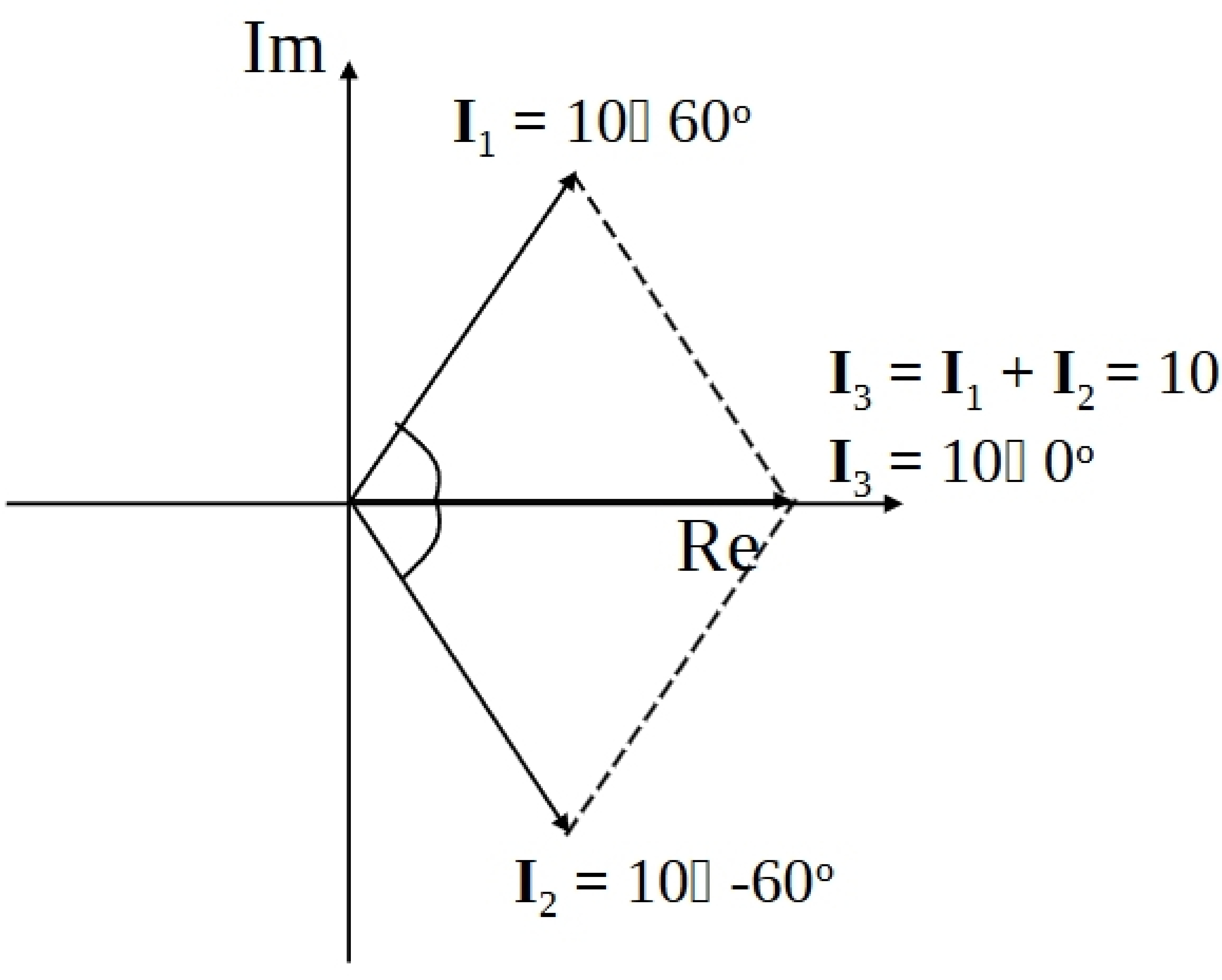
$$x_1(t) = A_1\cos(\omega t+\phi_1) \qquad x_2(t) = A_2\cos(\omega t+\phi_2)$$

$$\underline{X}_1 = A_1\angle\phi_1 \qquad \underline{X}_2 = A_2\angle\phi_2$$

$$x_1(t) + x_2(t) = \underline{X}_1 + \underline{X}_2$$

Example: KCL to find current I_3.





Multiplication of complex numbers

$$z = (a+jb)(c + jd) = (ac-bd) + j(ad+bc)$$

Multiplication of Phasors

$$\underline{X} = \underline{X}_1(j\omega) \text{ x } \underline{X}_2(j\omega) = (X_1\angle\phi_1)(X_2\angle\phi_2)$$

$$\underline{X} = X_1X_2\angle(\phi_1+\phi_2)$$

Example:

$$\underline{I} = \underline{Y}(j\omega)\,\underline{V} = (2.5\angle10^o)(3\angle26^o)$$

$$\underline{I} = 7.5\angle36^o \text{ A}$$

Division of complex numbers

$$z = \frac{(a+jb)}{(c+jd)} = \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)} = \frac{(ac+bd) + j(bc-ad)}{(c^2+d^2)}$$

Division of Phasors

$$\underline{X} = \underline{X}_1(j\omega) \div \underline{X}_2(j\omega) = (X_1\angle\phi_1)\div(X_2\angle\phi_2)$$

$$\underline{X} = \frac{X_1}{X_2}\angle(\phi_1-\phi_2)$$