LINEAR MOMENTUM AND COLLISIONS

CONSERVATION OF LINEAR MOMENTUM

Let's assume the collision of two objects (m_1, m_2), under the view of Newton's third law;

$$\vec{F}_{12} = -\vec{F}_{21} \longrightarrow \vec{F}_{12} + \vec{F}_{21} = 0$$

We know from Newton's second law that;

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

which is:

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

or:

$$\frac{d}{dt}\left(m_1 \vec{v}_1 + m_2 \vec{v}_2 \right) = 0$$

Therefore: $m_1 \vec{v}_1 + m_2 \vec{v}_2 = ct$

As we will see, in the absence of external forces, the linear momentum of a system is conserved!

$$\boxed{\vec{p} = m\vec{v}}$$ linear momentum.

Total (net) linear momentum:

$$\vec{P}_T \equiv \vec{P}_N = \sum_i \vec{P}_i = \sum_i m_i \vec{v}_i$$

$$\left.\begin{aligned} P_{Tx} &= \sum_i P_{ix} \\ P_{Ty} &= \sum_i P_{iy} \end{aligned}\right\} \quad \text{where} \quad \begin{aligned} P_x &= m v_x \\ P_y &= m v_y \end{aligned}$$

Newton's second law:

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt}}$$

Newton formulated it like that.

This is more general, since it allows to study the case in which mass changes with time.

Conservation of linear momentum:

$$\sum \vec{F} = \underbrace{\sum \vec{F}_{int}}_{\overset{\shortparallel}{0}} + \sum F_{ext} = \frac{d\vec{P}_T}{dt}$$

Newton's 3rd

Therefore; if $\sum F_{ext} = 0$ then;

$$\boxed{\vec{P}_T = \text{constant}}$$

IMPULSE AND MOMENTUM

As seen before, when a net force changes with time, there will be a change in linear momentum $\Delta\vec{p}$. This change is called 'Impulse' (I):

$$\boxed{\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}_N \, dt = \int_{t_i}^{t_f} \sum_i \vec{F}_i \, dt}$$

When working with average force $\left(\sum \vec{F}_{avg}\right)$, then

$$\boxed{\vec{I} = \Delta\vec{p} = \left(\sum F\right)_{avg} \Delta t}$$ where $\Delta t = t_f - t_i$

COLLISIONS IN ONE DIMENSION

Elastic collisions : $\Delta K = 0$, $\Delta\vec{p} = 0$

Inellastic collisions : $\Delta K \neq 0$ $\Delta\vec{p} = 0$

Perfectly inelastic collisions (when the objects stick together)

$V_{1f} = V_{2f} \equiv V$