HW12(a) Solution Hints <part 1>

The problems may not be identical to what I had assigned

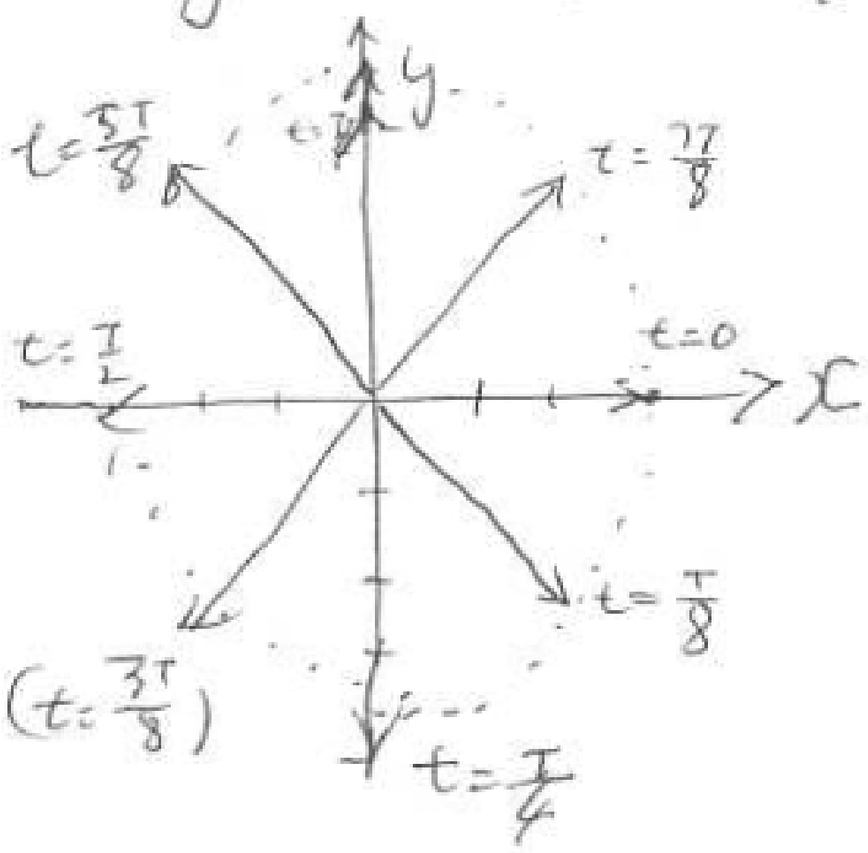
1. $\vec{E} = (3\hat{x} + 4j\hat{y})e^{-j\beta z}$

$\vec{E}(z,t) = Re[(3\hat{x}+4j\hat{y})e^{j\beta z}e^{j\omega t}]$

$= 3\hat{x}\cos(\omega t - \beta z) + 4\hat{y}\cos(\omega t - \beta z + 90^\circ)$

At $z=0$. $Ex(z,t) = 3\cos(\frac{2\pi}{T}t)$

$Ey(z,t) = 4\cos(\frac{2\pi}{T}t + 90^\circ) = -4\sin(\frac{2\pi}{T}t)$



2. (1) $\vec{E} = (2\hat{x} - 3\hat{y})e^{-j\beta z}$: LP (because $\phi_x = 0$, $\phi_y = a$, $\sin(\phi_x - \phi_y) = 0$)

(4) $\vec{E} = (2j\hat{x} - 3\hat{y})e^{-j\beta z}$: $\phi_x = 90^\circ$, $\phi_y = 180^\circ$, $\sin(\phi_x - \phi_y) = -1 < 0$, $a=2$ $b=3$, $a \neq b$: LHEP

(3) $\vec{E} = (2\hat{x} - 3j\hat{y})e^{-j\beta z}$

$\phi_x = 0$, $\phi_y = -90^\circ$, $\sin(0+90) = 1 > 0$, $a=2 \neq b=3$. RHEP

(4) $\vec{E} = (4\hat{x} + j4\hat{y})e^{-j\beta z}$

$\phi_x = 0$ $\phi_y = 90^\circ$ $\sin(\phi_x - \phi_y) = -1 < 0$

$a = b = 4$: LHCP

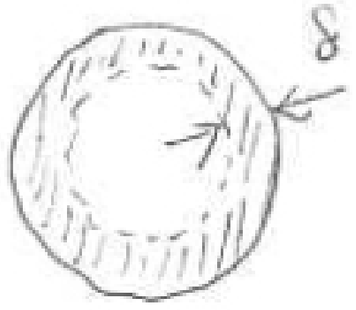
HW12 · part 2

8.4.9 $P_{rad} = Z_0 \frac{(kL)^2 I_{av}^2}{12\pi}$ eq. 8.23

at 1 G: $\lambda = 0.3\ m$, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.3}$, $Z_0 = 120\pi$

$$100\ (W) = 120\pi \cdot \frac{\left(\frac{2\pi}{0.3} \times 0.15\right)^2}{12\pi} \cdot I_{av}^2 \Rightarrow I_{av} = 1\ A$$

8.4.10



Good conductor, E field concentrated on "skin"

Cross section Area $\approx 2\pi a \delta$

$$R_L = \frac{L}{\sigma S} = \frac{L}{\sigma (2\pi a)\delta}$$

Skin depth $\delta = \frac{1}{\alpha}$.

Good Conductor $\alpha = \omega\sqrt{\mu\varepsilon}\left[\frac{1}{2}\sqrt{1+\tan^2\phi} - 1\right]^{1/2}$

$$\approx \omega\sqrt{\mu\varepsilon}\sqrt{\frac{\tan\phi}{2}} = \omega\sqrt{\mu\varepsilon}\sqrt{\frac{\sigma}{2\omega\varepsilon}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}$$

At 10 MHz: $f = 10^7\ Hz$, $\mu = \mu_0 = 4\pi \times 10^{-7}$ (H/m)

$\alpha = 4.78 \times 10^4$ $\alpha = \frac{1}{\delta} = 2 \times 10^{-5}$

$$R_L = \frac{L}{\sigma 2\pi a b} = \frac{0.2}{5.8\times10^7 \times 2\pi \times 10^{-3} \times 2\times10^{-5}} = 0.026\ \Omega$$

$$R_r = 80\pi^2\left(\frac{L}{\lambda}\right)^2 = 80\pi^2\left(\frac{0.2}{30}\right) = 0.035\ \Omega$$

$$\eta_r = \frac{R_r}{R_r + R_L} = 0.574 = 57.4\%$$