also been used to model a wide variety of other structures, from lightning strokes to helicopters and aircraft. Several structures that are in wide use (such as a coaxial cable, a strip line, and two parallel wires) can be modeled with a structure that consists of distributed inductors and capacitors. The direct application of Kirchhoff's laws leads to two first-order partial differential equations known as the telegraphers' equations. Eliminating one of the dependent variables between these two equations leads to a wave equation.

A sinusoidal signal generator will launch sinusoidal waves to increasing or decreasing values of the spatial coordinate assuming that the initial transient effects can be neglected. The amplitude of the wave will repeat itself every ***half wavelength***. The ratio of the voltage wave propagating in one direction to the current wave propagating in the same direction is the characteristic impedance of the transmission line. Terminating the transmission line with either a load impedance or another transmission line introduced the concepts of reflection and transmission coefficients, standing waves, the VSWR, and of impedance matching. The Smith chart is used to facilitate this matching. Transient effects and their subsequent propagation, along with pulse propagation, were analyzed with a bounce diagram. The final asymptotic state of a transmission line excited by a step voltage was found. Finally, the effects of loss and dispersion were analyzed.

7.12 Problems

7.1.1. Show that the equivalent circuit element parameters for the coaxial cable and the strip line are correct representations.

7.1.2. Show that the telegrapher's equations can be derived using an argument based on a Taylor series.

7.2.1. Show that the quantity $v = 1/\sqrt{\hat{L}\hat{C}}$ does indeed have the units of a velocity.

7.2.2. Show that the units of the diffusion coefficient $D = 1/\hat{R}\hat{C}$ do indeed have the units of (length)^2/time.

7.2.3. Show that a function which represents a wave that propagates to decreasing values of z satisfies the wave equation (7.7).

7.2.4. Let us replace the linear capacitors in Figure 7–3 with nonlinear varactor diodes whose capacitance depends on the voltage applied across them. In this case, the current ΔI into the diode can be written as $\Delta I = \partial Q(V)/\partial t$. Derive the resulting wave equation for this transmission line.

7.2.5. Show that the two equations given in (7.15) and (7.16) are equivalent, and find expressions for the constants in one equation in terms of the constants in the other.

7.3.1. Find an expression for the characteristic impedance of the strip line.

7.3.2. Find an expression for the characteristic impedance of a twin lead.

7.3.3. In an integrated circuit, a dielectric with $\varepsilon_r = 2$ is inserted between two metal conductors. The width of the top metal strip is 10 μm, and its separation from the bottom-grounded metal plane is 5 μm. Find the characteristic impedance of this transmission line and the velocity of a signal.

7.3.4. Design a strip line with a glass insulator that will have a characteristic impedance of 5 Ω. You will have some freedom in this design, but there is one constraint—it is to be used in an integrated circuit.

7.3.5. A TV twin lead consists of two parallel 1-mm copper wires separated by 1 cm of a rubber dielectric with $\varepsilon_r = 3$. What is the capacitance per meter of this twin lead? What is its characteristic impedance?

7.3.6. Prove that the voltage that appears across a load impedance will be less than the incident wave if $Z_L < Z_c$.

7.4.1. A VSWR is measured along a transmission line to be 2. Find two values for the reflection coefficient $\Re$. Which of these values will correspond to $Z_L < Z_c$ and which to $Z_L > Z_c$?

7.4.2. For Problem 7.4.1, find the two values of Z_L if $Z_c = 50\ \Omega$.

7.4.3. A load impedance $Z_L = 25\ \Omega$ is connected to a transmission line whose characteristic impedance is $50\ \Omega$. Using (7.33), plot the impedance as a function of a distance from the load to a total distance of 2λ.

7.5.1. Using (7.33), prove that the load impedance will repeat itself every $\lambda / 2$.

7.5.2. Using (7.33), prove that the input impedance of a transmission line terminated in a short circuit with a length $\mathcal{L}$, where $\lambda / 4 < \mathcal{L} < \lambda / 2$ is capacitive.

7.5.3. Using (7.33), prove that the input impedance of a transmission line terminated in an open circuit with a length $\mathcal{L}$, where $\lambda / 4 < \mathcal{L} < \lambda / 2$ is inductive.

7.5.4. An air-filled 50-Ω coaxial cable that is 1 m long is excited with a 300-MHz signal generator. The line is terminated with a load impedance $Z_L = (25 + j25)\ \Omega$. What is the input impedance of this line?

7.5.5. A shorted 50-Ω transmission line of length $\mathcal{L}$ has an input admittance of $-j\,0.01\ S$. Find the length of the line in λ.

7.5.6. Using (7.46) and (7.47), draw the circles for the values of $r = 0$, 1, and ∞ and $x = -1$, 0, and 1 to convince yourself that Figure 7–9 is correct.

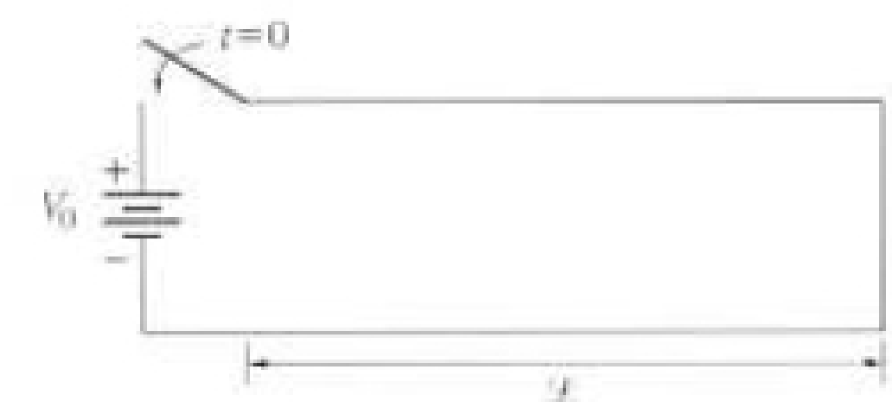
7.6.1. Using a Smith chart, find the impedance Z_{in} of a 50-Ω coaxial cable that is terminated in a load $Z_L = (25 + j25)\ \Omega$. The coaxial cable has a length of $3\lambda / 8$.

7.6.2. Using a Smith chart, find the admittance Y_{in} of a 50-Ω coaxial cable that is terminated in a load $Z_L = (25 + j25)\ \Omega$. The coaxial cable has a length of $\lambda / 8$.

7.6.3. Using a Smith chart, find the distance from a load impedance $Z_L = (25 + j25)\ \Omega$ that is connected to a 50-Ω coaxial cable where the normalized input admittance $Y_{in} = 1 + jB_{in}$. How long should a transmission line that is terminated in a short circuit be in order to match the transmission?

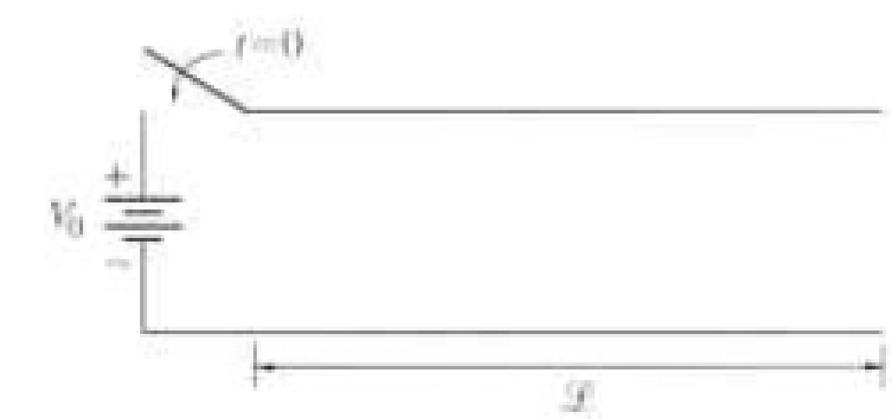
7.6.4. A load impedance $Z_L = (100 - j100)\ \Omega$ terminates a 50-Ω transmission line. Find the characteristic impedance of the quarter-wavelength matching transmission line.

7.6.5. A lossless battery is connected to an ideal transmission line with a characteristic impedance Z_c of length $\mathcal{L}$ that is terminated in a short circuit. Sketch the potential at $z = \mathcal{L} / 2$ as a function of time $0 < t < 4(\mathcal{L} / v)$. The switch is closed at $t = 0$.



7.7.1. Sketch the current profile at $z = \mathcal{L} / 2$ as a function of time $0 < t < 4(\mathcal{L} / v)$ at $z = \mathcal{L} / 2$ for the transmission line stated in Problem 7.6.5.

7.7.2. A lossless battery is connected to an ideal transmission line with a characteristic impedance Z_c of length $\mathcal{L}$ that is terminated in an open circuit. Sketch the potential at $z = \mathcal{L} / 2$ as a function of time $0 < t < 4(\mathcal{L} / v)$. The switch is closed at $t = 0$.



7.7.3. Sketch the current profile at $z = \mathcal{L} / 2$ as a function of time $0 < t < 4(\mathcal{L} / v)$ for the transmission line stated in Problem 7.7.2.

7.7.4. Two transmission lines are joined with a resistor R_L.

Show that the transmitted voltage V_T in Line 2 can be written as

$$V_T = \frac{Z_{c2}}{R_L + Z_{c2}}(1 + \Re)V_{inc}$$

where

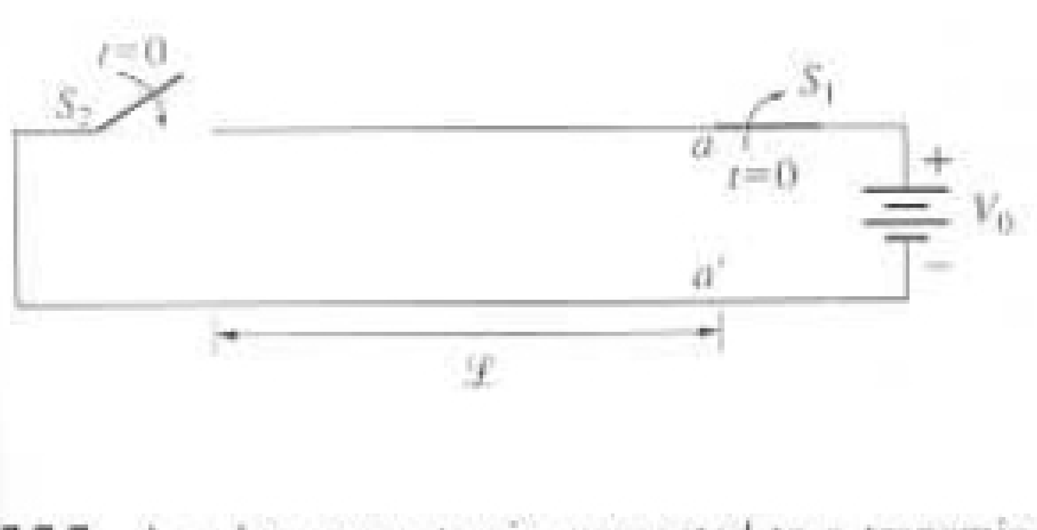
$$\Re = (R_L + Z_{c2} - Z_{c1})/(R_L + Z_{c2} + Z_{c1})$$

if the voltage incident from $z = -\infty$ is V_{inc}.

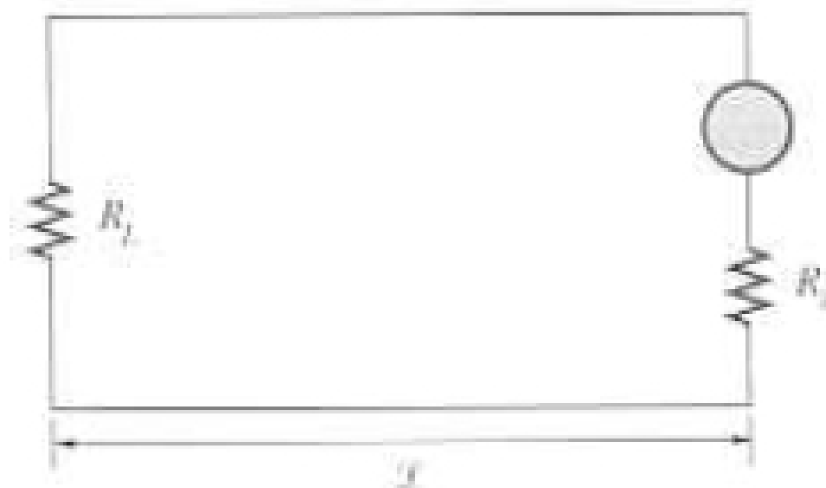
7.7.5. At $t = 0$, the switch located at the load is closed. Sketch the voltage and the current at the load as a function of time $\mathcal{L}/v$. The impedance at the load R_L = the characteristic impedance of the line Z_c.

7.7.6. A transmission line with two switches, one at the battery and one at the load, is shown below. Initially, switch S_1 is closed and switch S_2 is open. At $t = 0$, S_1 is opened and S_2 is closed. Sketch the voltage $V_{aa'}$ as a function of time.

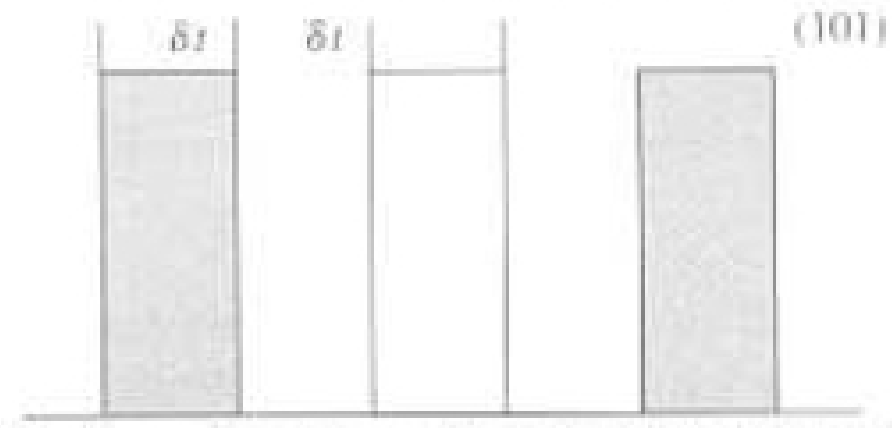
If there were a load impedance located at the midpoint of this particular transmission line, it would be possible to generate a large voltage pulse across this load impedance. This is called a ***Blumlein transmission line***.



7.7.7. A pulse generator is connected to a transmission line of length $\mathcal{L} = 2$ m having $Z_c = 50\ \Omega$, $R_L = 20\ \Omega$, and $R_g = 30\ \Omega$. The propagation velocity in this transmission line is equal to 10^8 m/s. The amplitude of the pulse is 1 V and its width is 10^{-9} s. Plot the voltage at $z = \mathcal{L}/2$ as a function of time, $0 \le t \le 100$ ns.



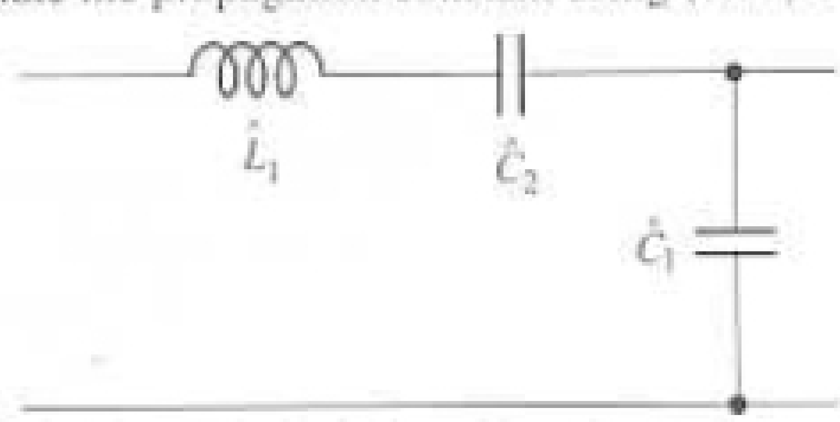
7.8.1. In a digital computer we want to transmit a sequence of binary pulses from a pulse generator to another point where they are to be sampled. Let us assume that the pulse sequence shown below is launched by the pulse generator shown in Problem 7.7.7, and we want to detect the sequence at a time $T = 3\mathcal{L}/2v$ later. Describe any limitations that might be imposed on the speed of this computer.



7.8.2. Generalize the results of Problem 7.8.1 to 32-bit and 64-bit machines.

7.8.3. Repeat Problem 7.7.7 if the transmission line is lossy and the signal decays as $e^{-\alpha z}$ as it propagates, where $\alpha = 0.01, 0.1$, and 1.

7.9.1. Sketch the dispersion relation for a transmission line consisting of a series-resonant circuit in the series branch and a capacitor in the shunt branch. Calculate the propagation constant using (7.77).



7.9.2. Describe the dispersion relation for a transmission line consisting of an inductor in the series branch and a tank circuit in the shunt branch.