Example

6/30

Notes
Sec 6.1 Ex 4
" Ex 5
" Prob 31
" Ex 7
" Prob 6

Memorize — Table 6.2.1

$f(t)$	$F(s) = \mathcal{L}(f(t))$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n, n=integer	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$

$\mathcal{L}(y) = \frac{4}{s-3} \Rightarrow 4\left(\frac{1}{s-3}\right) = 4e^{3t}$

$\mathcal{L}(y) = \frac{9}{s^2+9} \Rightarrow 3\left(\frac{3}{s^2+9}\right) = 3\sin 3t$

$\mathcal{L}(y) = \frac{4}{s^2+25} \Rightarrow y = x\left(\frac{5}{s^2+25}\right) \Rightarrow 5x = 4 \quad x = 4/5$

$\frac{4}{5}\sin(5t) \quad \dots \quad y^{n-1}(0)$

$y'' - 5y' + 6y = 0 \quad y(0) = 1 \quad y'(0) = 4$

$r = 2, 3$

$y = c_1 e^{2x} + c_2 e^{3x}$

$y' = 2c_1 e^{2x} + 3c_2 e^{3x}$

$1 = c_1 + c_2$

$4 = 2c_1 + 3c_2$

$-2 = -2c_1 - 2c_2$

$4 = 2c_1 + 3c_2$

$\Rightarrow 2 = c_2$

$-1 = c_1$

$\mathcal{L}(y'' - 5y' + 6y = 0) \Rightarrow \mathcal{L}(y'') - 5\mathcal{L}(y') + 6\mathcal{L}(y) = 0$

$\Rightarrow s^2\mathcal{L}(y) - s\,y(0) - y'(0) - 5(s\mathcal{L}(y) - y(0)) + 6\mathcal{L}(y)$

$\mathcal{L}(y)(s^2 - 5s + 6) - s - 4 + 5 = 0$

$\mathcal{L}(y) = \frac{s-1}{s^2-5s+6} = \frac{a}{s-2} + \frac{b}{s-3} \Rightarrow \frac{(s-3)a}{(s-2)(s-3)} + \frac{b(s-2)}{(s-2)(s-3)}$

$a(s-3) + b(s-2) = s-1$

$(a+b)s + (-3a - 2b) = s - 1$

$a + b = 1 \quad -3a - 2b = -1$

$3a + 3b = 3$

$-3a - 2b = -1$

$b = 2$

$a = -1$

$\Rightarrow -1\left(\frac{1}{s-2}\right) + 2\left(\frac{1}{s-3}\right) = -1e^{2t} + 2e^{3t}$

6/30

$\mathcal{L}(y'' - 8y' + 12y = 3)$ $y(0) = 0$ $y'(0) = 3$

$\mathcal{L}(y'') - 8\mathcal{L}(y') + 12\mathcal{L}(y) = \mathcal{L}(3)$

$= s^2\mathcal{L}(y) - sy(0) - y'(0) - 8(s\mathcal{L}(y) - y(0)) + 12\mathcal{L}(y) = \frac{3}{s}$

$\mathcal{L}(y)(s^2 - 8s + 12) - 3 = \frac{3}{s}$

$\mathcal{L}(y) = \frac{\frac{3}{s} + 3}{s^2 - 8s + 12} = \frac{3s+3}{s(s-6)(s-2)} \Rightarrow \frac{a}{s} + \frac{b}{s-2} + \frac{c}{s-6}$

$a((s-2)(s-6)) + b(s(s-6)) + c(s(s-2)) = 3s + 3$

$a(s^2 - 8s + 12) + b(s^2 - 6s) + c(s^2 - 2s) = 3s + 3$

$= (a+b+c)(s^2) + s(-8a - 6b - 2c) + (12) = 3s + 3$

$a + b + c = 0$ $\quad b + c = -\frac{1}{4}$ $\quad 2b + 2c = -\frac{1}{2}$

$-8a - 6b - 2c = 3$ $\quad -6b - 2c = 5$ $\quad -6b - 2c = 5$

$12a = 3$ $\quad a = \frac{1}{4}$ $\quad -4b = \frac{9}{2}$

$b = \frac{-9}{8}$

$c = -\frac{1}{4} + \frac{9}{8} = \frac{7}{8} = c$

$\frac{1}{4}\left(\frac{1}{s}\right) - \frac{9}{8}\left(\frac{1}{s-2}\right) + \frac{7}{8}\left(\frac{1}{s-6}\right) = \frac{1}{4} - \frac{9}{8}(e^{2t}) + \frac{7}{8}e^{6t}$

HW: 6.2 1-15 odd, 19.

9) $e^{at}\sin bt \Rightarrow \mathcal{L} = \frac{b}{(s-a)^2 + b^2}$

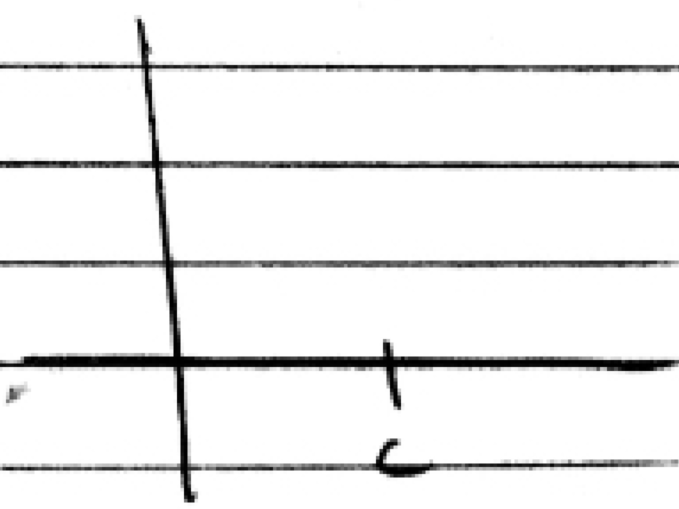
10) $e^{at}\cos bt \Rightarrow \mathcal{L} = \frac{s-a}{(s-a)^2 + b^2}$

Step function or Heaviside

7/1

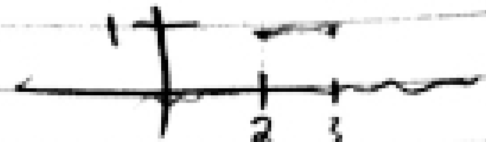
$u_c(t) = \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t \geq c \end{cases}$

$1 - u_c(t) = \begin{cases} 1 & \text{if } t < 0 \\ 0 & \text{if } t \geq 0 \end{cases}$



7/1

$$u_2(t) - u_3(t) = \begin{cases} 0 & \text{if } t < 2 \\ 1 & \text{if } 2 \le t < 3 \\ 0 & \text{if } t \ge 3 \end{cases}$$

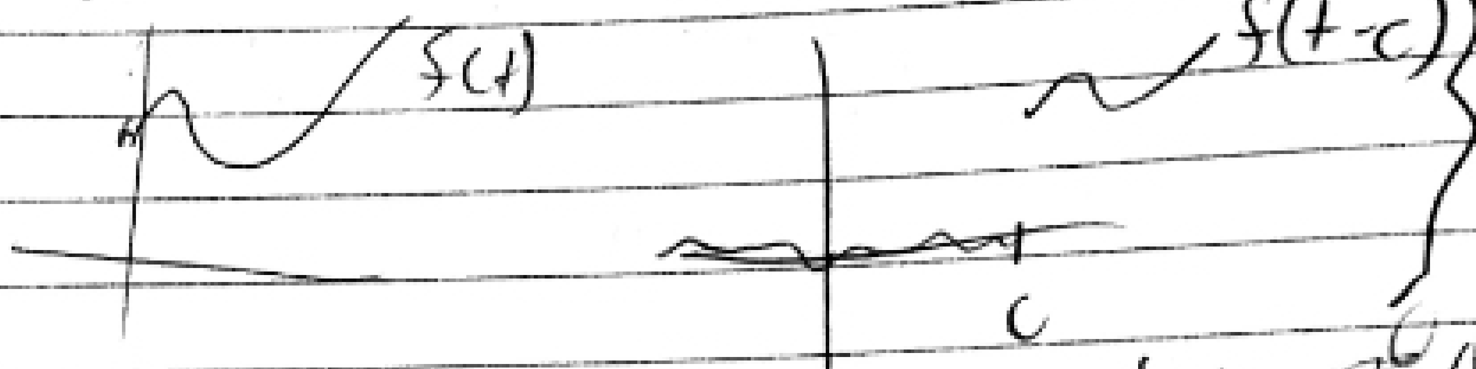


$$f(t) = \begin{cases} 1 & \text{if } t < 3 \\ 4 & \text{if } 3 \le t < 5 \\ -3 & \text{if } 5 \le t < 7 \\ 10 & \text{if } 7 \le t \end{cases}$$

$$g(t) = 1 + 3u_3(t) - 7u_5(t) + 13u_7(t)$$

$$f(t) = \begin{cases} 2 & \text{if } t < 1 \\ 3 & \text{if } 1 \le t < 4 \\ -5 & \text{if } 4 \le t < 6 \\ 20 & \text{if } 6 \le T \end{cases}$$

$$g(t) = 2 + 1u_1(t) - 8u_4(t) + 25u_6(t)$$



$u_c(t)f(t-c)$

$$\mathcal{L}(u_c(t)) = \int_0^\infty e^{-st} u_c(t) = \cancel{\int_0^c e^{-st} u_c(t)} + \int_c^\infty e^{-st} u_c(t)$$

$$\Rightarrow \int_c^\infty e^{-st} = -\frac{e^{-st}}{s}\Big|_c^A = \lim_{A\to\infty} \cancel{\frac{-e^{-sA}}{s}} + \frac{e^{-sc}}{s} = \frac{e^{-sc}}{s}$$

$$\mathcal{L}(u_c(t)) = \frac{e^{-sc}}{s}$$