Today's topics

- Counting
  - Generalized Pigeonhole Principle
  - Permutations
  - Combinations
  - Binomial Coefficients
  - Writing permutation algorithms
- Reading: Sections 4.2-4.3, 4.4, 4.6
- Upcoming
  - Probability

Generalized Pigeonhole Principle

- If N objects are assigned to k places, then at least one place must be assigned at least $\lceil N/k \rceil$ objects.
- *E.g.*, there are N=280 students in this class. There are k=52 weeks in the year.
  - Therefore, there must be at least 1 week during which at least $\lceil 280/52 \rceil = \lceil 5.38 \rceil = 6$ students in the class have a birthday.

Proof of G.P.P.

- By contradiction. Suppose every place has $< \lceil N/k \rceil$ objects, thus $\leq \lceil N/k \rceil - 1$.
- Then the total number of objects is at most

$$k\left(\left\lceil \frac{N}{k} \right\rceil - 1\right) < k\left(\left(\frac{N}{k} + 1\right) - 1\right) = k\left(\frac{N}{k}\right) = N$$

- So, there are less than N objects, which contradicts our assumption of N objects! □