

# Today's topics

- Integers & Number Theory
  - Integers
  - Division, GCD
  - Euclidean Alg
  - Mod!
- Reading: Sections 2.4-2.6
- Upcoming
  - Sequences, Summations, & Induction

# Forward Reasoning

- Have premises  $p$ , and want to prove  $q$ .
  - Find a  $s_1$  such that  $p \rightarrow s_1$ 
    - Then, modus ponens gives you  $s_1$ .
  - Then, find an  $s_2 \ni$  (such that)  $s_1 \rightarrow s_2$ .
    - Then, modus ponens gives you  $s_2$ .
  - And hope to eventually get to an  $s_n \ni s_n \rightarrow q$ .
- The problem with this method is...
  - It can be tough to see the path looking from  $p$ .

# Backward Reasoning

- It can often be easier to see the *very same path* if you just start looking from the conclusion  $q$  instead...
  - That is, first find an  $s_{-1}$  such that  $s_{-1} \rightarrow q$ .
  - Then, find an  $s_{-2} \ni s_{-2} \rightarrow s_{-1}$ , and so on...
  - Working back to an  $s_{-n} \ni p \rightarrow s_{-n}$ .
- Note we *still* are using *modus ponens* to propagate truth *forwards* down the chain from  $p$  to  $s_{-n}$  to ... to  $s_1$  to  $q$ !
  - We are *finding* the chain *backwards*, but *applying* it *forwards*.
  - This is not quite the same thing as an indirect proof...
    - In that, we would use *modus tollens* and  $\neg q$  to prove  $\neg s_{-1}$ , etc.
  - However, it is similar.