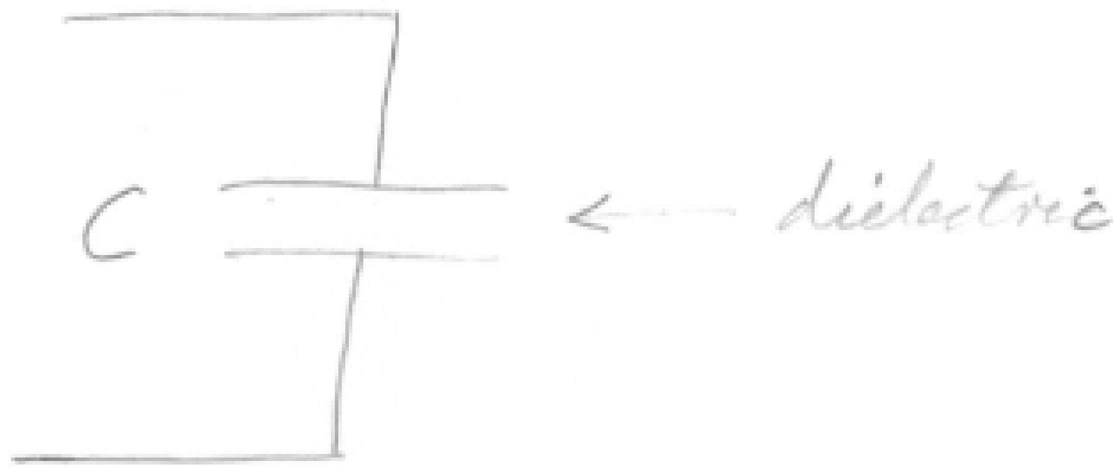
A. Capacitor

6~1

1. Two conductors separated by an insulator



2. Terminal characteristics

a) $+\Delta q$ from bottom to top

b) requires Δv

c) specifically:

$$\Delta q = C \,\Delta v \qquad \text{(linear relation)}$$

or $\boxed{q = Cv}$ C is capacitance in Farads

d) requires time Δt

$$\therefore \frac{\Delta q}{\Delta t} = C \frac{\Delta v}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \lim_{\Delta t \to 0} C \frac{\Delta v}{\Delta t}$$

$$\frac{dq}{dt} = C \frac{dv}{dt}$$

or $\boxed{i = C \frac{dv}{dt}}$

$$C = \frac{q}{v}$$

$$C = \frac{\epsilon A}{d}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$

Permitivity

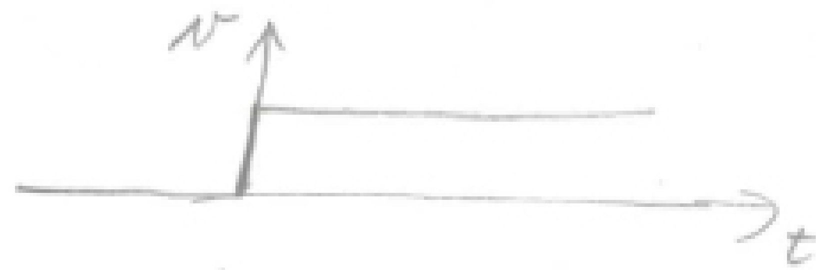
3. Additional properties

a) since $i = C\frac{dv}{dt}$

b) $i = 0$ for constant v

c) $\therefore$ open circuit to d.c. (for steady state)

d) Also, the voltage across a capacitor cannot change instantaneously



$$\frac{dv}{dt} = \infty$$

$\therefore i = C\frac{dv}{dt} = \infty$ would require an ∞ current

e) voltage in terms of the current

$$dv = \frac{1}{C}\, i\, dt$$

$$\int_{v(t_0)}^{v(t)} dv = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau$$

$$\therefore \boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)}$$

initial voltage ($v(t_0)$)

f) Energy relation

$$w(t) = \int_{-\infty}^{t} v\, i\, d\tau$$

$$= \int_{-\infty}^{t} v \left(C \frac{dv}{d\tau} \right) d\tau$$

$$= C \int_{v(-\infty)}^{v(t)} v\, dv$$

$$= \frac{1}{2} C v^2 \Big|_{v(-\infty)}^{v(t)}$$

$$\boxed{w(t) = \frac{1}{2} C v^2(t)}$$

stored in an electric field

$\therefore \quad w \geq 0$ (capacitor is a passive element)