BIOS 7250 - Chapter 11 – Probability Proportional to Size Sampling:

I. PPS Sampling With Replacement

A. An unequal sampling plan, where the probability of a unit being selected is proportionate to size of the cluster the unit belongs to.

B. Sampling Plan:

1. Stage I: Sample m clusters with replacement. The probability of selection is $\pi' = \frac{N_i}{N}$.
2. Stage II: Select a simple random sample of $\bar{n}$ enumeration units from each sampled cluster (clusters can repeat).

C. Hansen-Hurwitz Estimator of Population Total, Y:

$$y'_{ppswr} = \frac{N}{n}\sum_{i=1}^{m} y_i = \frac{N}{n}\sum_{i=1}^{m}\sum_{j=1}^{\bar{n}} y_{ij}, \qquad (11.9)$$

where $N = \sum_{i=1}^{M} N_i$ and $n = \bar{n}m$,

D. Estimated Standard Error for y'_{ppswr}, when $\pi' = \frac{N_i}{N}$:

$$\hat{SE}(y_{ppswr}) = \sqrt{\frac{\sum_{i=1}^{m}\left(\frac{Ny_i}{\bar{n}} - y'_{ppswr}\right)^2}{m(m-1)}} \qquad (11.11)$$

E. PPS sampling plan where the probability of a unit being selected is proportionate to some known variable, X.

1. Hansen-Hurwitz Estimator of Population Total, Y:

$$y_{ppswr}' = \frac{1}{m}\sum_{i=1}^{m}\frac{N_i}{n\pi_i'}y_i = \frac{1}{m}\sum_{i=1}^{m}\frac{N_i}{n\pi_i'}\sum_{j=1}^{\bar{n}}y_{ij}, \qquad (11.8)$$

where $\pi_i' = \frac{X_i}{X}$

2. Estimated Standard Error for y_{ppswr}', when $\pi' = \frac{X_i}{X}$:

$$\hat{SE}(y_{ppswr}') = \sqrt{\frac{\sum_{i=1}^{m}\left(\frac{NXy_i}{n\bar{X}_i} - y_{ppswr}'\right)^2}{m(m-1)}} \qquad (11.10)$$

3. When $X_i = N_i$, (11.8) → (11.9) and (11.10) → (11.11)

F. Example: Page 344.

II. How to Take a PPS Sample with Replacement when N_i is the measure of size variable.

A. Section 11.3.3, page 353, using Table 10.9, page 343. Assume m clusters are to be sampled with replacement, and $\bar{n}$ enumeration units are to be sampled at each drawing of a cluster.

1. Cumulate the measure of size variable, N_i.
2. Associate random variables with each cluster, based on the cumulative measure. (See Table 11.8, page 357)
3. Take a random number between 1 and N, and select the cluster corresponding to the random number.
4. Take a simple random sample of $\bar{n}$ enumeration units without replacement from the cluster selected in step 3.
5. Repeat steps 3 and 4 until m clusters and $n = \bar{n}m$ enumeration units are chosen.

B. Example: Table 11.8, Page 357.