P4.7 Consider a sphere of radius R immersed in a uniform stream U_0, as shown in Fig. P4.7. According to the theory of Chap. 8, the fluid velocity along streamline AB is given by

$$\mathbf{V} = u\mathbf{i} = U_o\left(1 + \frac{R^3}{x^3}\right)\mathbf{i}$$

Find (a) the position of maximum fluid acceleration along AB and (b) the time required for a fluid particle to travel from A to B. Note that x is negative along line AB.

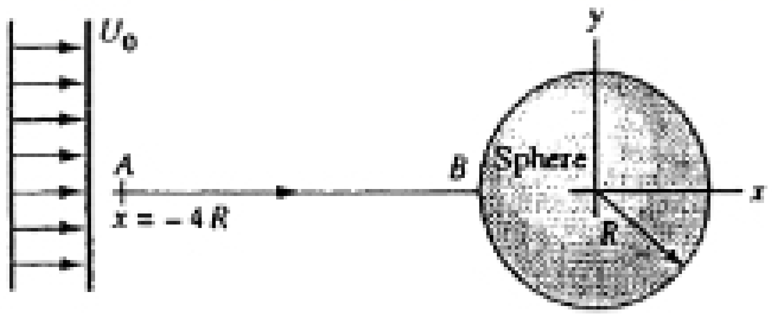


Fig. P4.7

Solution: (a) Along this streamline, the fluid acceleration is one-dimensional:

$$a_x = u\frac{\partial u}{\partial x} = U_o(1 + R^3/x^3)(-3U_oR^3/x^4) = -3U_o^2R^3(x^{-4} + R^3x^{-7}) \quad \text{for } x \le -R$$

The maximum occurs where $d(a_x)/dx = 0$, or at $x = -(7R^3/4)^{1/3} \approx \mathbf{-1.205R}$ *Ans.* (a)
(b) The time required to move along this path from A to B is computed from

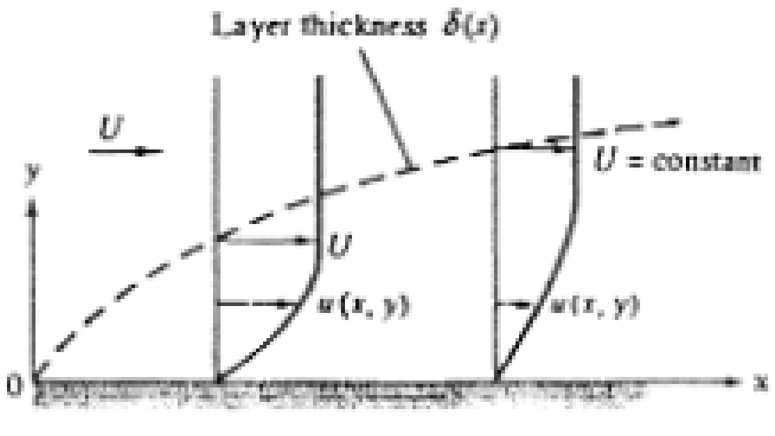
$$u = \frac{dx}{dt} = U_o(1 + R^3/x^3), \quad \text{or:} \quad \int_{-4R}^{-R} \frac{dx}{1 + R^3/x^3} = \int_0^t U_o\,dt,$$

$$\text{or:} \quad U_o t = \left[x - \frac{R}{6}\ln\frac{(x+R)^2}{x^2 - Rx + R^2} + \frac{R}{\sqrt{3}}\tan^{-1}\left(\frac{2x - R}{R\sqrt{3}}\right)\right]_{-4R}^{-R} = \infty$$

It takes **an infinite time** to actually *reach* the stagnation point, where the velocity is zero. *Ans.* (b)

P4.17 An excellent approximation for the two-dimensional incompressible laminar boundary layer on the flat surface in Fig. P4.17 is

$$u \approx U\left(2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right) \text{ for } y \le \delta \text{, where } \delta = Cx^{1/2},\ C = \text{ constant}$$



(*a*) Assuming a no-slip condition at the wall, find an expression for the velocity component $v(x, y)$ for $y \le \delta$. (*b*) Then find the maximum value of v at the station $x = 1$ m, for the particular case of airflow, when $U = 3$ m/s and $\delta = 1.1$ cm.

Solution: (*a*) With u known, use the two-dimensional equation of continuity to find v:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -U\left(-\frac{2y}{\delta^2}\frac{d\delta}{dx} + \frac{6y^3}{\delta^4}\frac{d\delta}{dx} - \frac{4y^4}{\delta^5}\frac{d\delta}{dx}\right),$$

$$or: \; v = 2U\frac{d\delta}{dx}\int_0^y \left(\frac{y}{\delta^2} - \frac{3y^3}{\delta^4} + \frac{2y^4}{\delta^5}\right)dy = 2U\frac{d\delta}{dx}\left(\frac{y^2}{2\delta^2} - \frac{3y^4}{4\delta^4} + \frac{2y^5}{5\delta^5}\right) \qquad Ans.(a)$$

(*b*) First evaluate C from the given data at $x = 1$ m:

$$\delta = 0.011m = C(1m)^{1/2}, \quad \text{hence } C = 0.011\, m^{1/2}$$

$$\text{Or, alternately}, \; \frac{d\delta}{dx} = \frac{1}{2}Cx^{-1/2} = \frac{1}{2}\left(\frac{\delta}{x^{1/2}}\right)x^{-1/2} = \frac{\delta}{2x}$$

Substitute this into *Ans*.(*a*) above and note that v rises monotonically with y to a **maximum** at the outer edge of the boundary layer, $y = \delta$. The maximum velocity v is thus

$$v_{\max} \approx 2U\frac{d\delta}{dx}\left(\frac{1}{2} - \frac{3}{4} + \frac{2}{5}\right) = 2\left(3\frac{m}{s}\right)\left[\frac{0.011m}{2(1m)}\right]\left(\frac{3}{20}\right) \approx \mathbf{0.0050\ \frac{m}{s}} \quad Ans.(b)$$

This is slightly smaller than the exact value of $v_{\max}$ from laminar boundary theory (Chap. 7).

P4.31 According to potential theory (Chap. 8) for the flow approaching a rounded two-dimensional body, as in Fig. P4.31, the velocity approaching the stagnation point is given by $u = U(1 - a^2/x^2)$, where a is the nose radius and U is the velocity far upstream. Compute the value and position of the maximum viscous normal stress along this streamline. Is this also the position of maximum fluid deceleration? Evaluate the maximum viscous normal stress if the fluid is SAE 30 oil at 20°C, with $U = 2$ m/s and $a = 6$ cm.

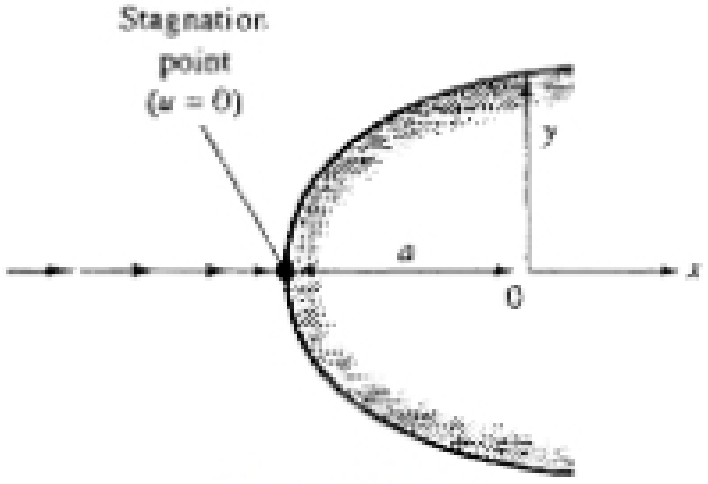


Fig. P4.31

Solution: (a) Along this line of symmetry the convective deceleration is one-dimensional:

$$a_x = u\frac{\partial u}{\partial x} = U\left(1-\frac{a^2}{x^2}\right)U\left(\frac{2a^2}{x^3}\right) = 2U^2\left(\frac{a^3}{x^3}-\frac{a^4}{x^5}\right)$$

This has a maximum deceleration at $\frac{da_x}{dx} = 0$, or at $\mathbf{x = -\sqrt{(5/3)}\,a = -1.29a}$ *Ans.* (a)

The value of maximum deceleration at this point is $a_{x,max} = -0.372U^2/a$.

(b) The viscous normal stress along this line is given by

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} = 2\mu\left(\frac{2a^2U}{x^3}\right) \text{ with a maximum } \boldsymbol{\tau_{max} = \frac{4\mu U}{a}} \textbf{ at } \mathbf{x = -a} \quad \textit{Ans. (b)}$$

Thus maximum stress does not occur at the same position as maximum deceleration. For SAE 30 oil at 20°C, we obtain the numerical result

$$\text{SAE 30 oil,} \quad \rho = 917\ \frac{kg}{m^3}, \quad \mu = 0.29\ \frac{kg}{m\cdot s}, \quad \tau_{max} = \frac{4(0.29)(2.0)}{(0.06\text{ m})} \approx \mathbf{39\ Pa} \quad \textit{Ans. (b)}$$

P4.41 As mentioned in Sec. 4.10, the velocity profile for laminar flow between two plates, as in Fig. P4.40, is

$$u = \frac{4u_{max}y(h-y)}{h^2} \qquad \upsilon = w = 0$$

If the wall temperature is *Tw* at both walls, use the incompressible-flow energy equation (4.75) to solve for the temperature distribution *T*(*y*) between the walls for steady flow.

Solution: Assume T = T(y) and use the energy equation with the known u(y):

$$\rho c_p\frac{dT}{dt} = k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2, \quad \text{or:} \quad \rho c_p(0) = k\frac{d^2T}{dy^2} + \mu\left[\frac{4u_{max}}{h^2}(h-2y)\right]^2, \quad \text{or:}$$

$$\frac{d^2T}{dy^2} = -\frac{16\mu u_{max}^2}{kh^4}(h^2 - 4hy + 4y^2), \quad \text{Integrate:} \quad \frac{dT}{dy} = \frac{-16\mu u_{max}^2}{kh^4}\left(h^2y - 2hy^2 + \frac{4y^3}{3} + C_1\right)$$