

Methods for combining functions:

1. Sum  $(f + g)(x) = f(x) + g(x)$
2. Difference  $(f - g)(x) = f(x) - g(x)$
3. Product  $(fg)(x) = f(x)g(x)$
4. Quotient  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$  provided  $g(x) \neq 0$ .
5. Composition  $(f \circ g)(x) = f(g(x))$

## Example 1:

Let  $f(x) = x^2 + 5$  and  $g(x) = x + 5$ .

a. Find  $f + g = x^2 + 5 + x + 5 = x^2 + x + 10$

b. Find  $f - g = x^2 + 5 - (x + 5) = x^2 + 5 - x - 5 = x^2 - x$

c. Find  $f \cdot g = (x^2 + 5)(x + 5) = x^3 + 5x^2 + 5x + 25$

d. Find  $f / g = \frac{x^2 + 5}{x + 5} \quad x \neq -5$

e. Find  $f(-1) - g(2) = 6 - 7 = -1$   
 $f(x) = x^2 + 5$   
 $f(-1) = (-1)^2 + 5 = 1 + 5 = 6$   
 $g(x) = x + 5$   
 $g(2) = 2 + 5 = 7$

**Example 2:** Suppose  $f(x) = 2x - 3$ ,  $g(x) = x^2 - 4x + 5$ , and  $h(x) = 2x^3$ .

$$\text{a. } (f - g)(x) = 2x - 3 - (x^2 - 4x + 5) \\ = 2x - 3 - x^2 + 4x - 5 = -x^2 + 6x - 8$$

$$\text{b. } (f + g)(2) = f(2) + g(2) = 1 + 1 = 2 \\ f(2) = 2(2) - 3 = 4 - 3 = 1 \\ g(2) = (2)^2 - 4(2) + 5 = 9 - 8 = 1$$

$$\text{c. } \left(\frac{g}{h}\right)(x) = \frac{x^2 - 4x + 5}{2x^3} \quad x \neq 0$$

**Composition of Functions:**

Machine picture:  $(f \circ g)(x) = f(g(x))$

The diagram illustrates the composition of functions. It starts with an input  $x$  on the left. An arrow points from  $x$  to a box labeled  $g$ . From the box  $g$ , an arrow points to the expression  $g(x)$ . From  $g(x)$ , another arrow points to a second box labeled  $f$ . Finally, an arrow points from the box  $f$  to the expression  $f(g(x))$ .

This combined "machine" is called  $f \circ g$  (read " $f$  composed with  $g$ ").

The new function  $f \circ g$  is defined whenever  $x$  is in the domain of  $g$  and  $g(x)$  is in the domain of  $f$ .

$(f \circ g)(x) = f(g(x))$  the second function is put into the first function.

**Example 3:**

Let  $f(x) = x^2$  and  $g(x) = x - 3$

Find

$$\text{a. } (f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 \\ = (x-3)(x-3) = x^2 - 3x - 3x + 9 = x^2 - 6x + 9$$

$$\text{b. } (g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3$$

$$\text{c. } (f \circ g)(5) = f(g(5)) = f(2) = (2)^2 = 4 \\ g(5) = 5 - 3 = 2$$

**Example 4:**

Let  $f(x) = 2x + 3$  and  $g(x) = 4x - 1$

Find

$$\text{a. } (f \circ g)(x) = f(g(x)) = f(4x-1) = 2(4x-1) + 3 \\ = 8x - 2 + 3 = 8x + 1$$

$$\text{b. } (f \circ f)(x) = f(f(x)) = f(2x+3) = 2(2x+3) + 3 \\ = 4x + 6 + 3 = 4x + 9$$

$$\text{c. } (g \circ g)(-1) = g(g(-1)) = g(-5) = 4(-5) - 1 = -20 - 1 = -21 \\ g(-1) = 4(-1) - 1 = -4 - 1 = -5$$