ME451
Kinematics and Dynamics of Machine Systems

Dynamics of Planar Systems

December 9, 2010

Solving Index 3 DAEs using Newmark Method

Before we get started…

- Last Time:
  - Finish the discussion started last time
    - Marked the end of the Dynamics Analysis of Mechanical Systems chapter
- Today
  - Discuss the final exam
  - Inverse Dynamics Analysis
  - Equilibrium Analysis
  - We hug each other and stuff
- Final Exam
  - Tu, Dec. 21 at 5:05 PM, Room: **ME1245**

Done once, at the very beginning of the dynamics simulation

Find initial conditions for general coordinate positions and velocities that satisfy position and velocity constraint equations

↓

Use Newton's equations of motion along with the acceleration constraint equations to find general coordinate accelerations and lambdas at time = 0.

$$\begin{bmatrix} \mathbf{M} & \Phi_{\mathbf{q}}^T \\ \Phi_{\mathbf{q}} & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^A \\ \gamma \end{bmatrix}$$

At each time step (solving for the state of the system at t_{n+1}…)

Increment $t_{n+1} = t_n + h$
Define initial guess for accelerations q_dotdot and lambdas (take values from previous time step t_n)

↓

Iterate to solve nonlinear system obtained after Newmark discretization of DAEs.

Update t_{n+1} position and velocity using Newmark's formulas and the most recent values for acceleration and lambdas.

$$\begin{cases} \mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + \frac{h^2}{2}\left[(1-2\beta)\,\ddot{\mathbf{q}}_n + 2\beta\ddot{\mathbf{q}}_{n+1}\right] \\ \dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\left[(1-\gamma)\,\ddot{\mathbf{q}}_n + \gamma\ddot{\mathbf{q}}_{n+1}\right] \end{cases}$$

Get Jacobian of discretized equation. Use most recent q, q_dot, q_dotdot, and lambda at n+1

$$\hat{\mathbf{J}} = \begin{bmatrix} \mathbf{M} & \Phi_{\mathbf{q}}^T \\ \Phi_{\mathbf{q}} & 0 \end{bmatrix}$$

Use most recent values at n+1 to compute what each equation equals. Each equation will probably not equal zero. These values are called the residuals.

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \Phi_{\mathbf{q}}^T(\mathbf{q})\lambda - \mathbf{Q}^A(\dot{\mathbf{q}}, \mathbf{q}, t) = \mathbf{0} \\ \frac{1}{\beta h^2}\Phi(\mathbf{q}, t) = \mathbf{0} \end{cases}$$

Compute the correction vector.

$$\begin{bmatrix} \delta\ddot{\mathbf{q}} \\ \delta\lambda \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \Phi_{\mathbf{q}}^T \\ \Phi_{\mathbf{q}} & 0 \end{bmatrix}^{-1} \cdot \Psi(\ddot{\mathbf{q}}^{(old)}, \lambda^{(old)})$$

residual

Correct the most recent acceleration and lambda to get better approximations for accelerations and lambdas.

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix}^{(new)} = \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix}^{(old)} - \begin{bmatrix} \delta\ddot{\mathbf{q}} \\ \delta\lambda \end{bmatrix}$$

Compute the largest value present in the correction factor matrix and compare to a specified limit. This is the convergence check (largest correction is easily computed as infinity norm in MATLAB)

Is $\|\delta\ddot{\mathbf{q}}\| > 10^{-2}$?
(You might have to play with the value 10^{-2} a bit. 10^{-2} is just a good rule of thumb.)

Yes. Need to refine current acc/Lagr. Multiplier values

No.

Store acceleration and lambda for time t_{n+1}. Use this accelerations to compute the position and velocity at t_{n+1} and store these quantities as well.