A Review of Complex Arithmetic

A **complex** value C has both a **real** and **imaginary** component:

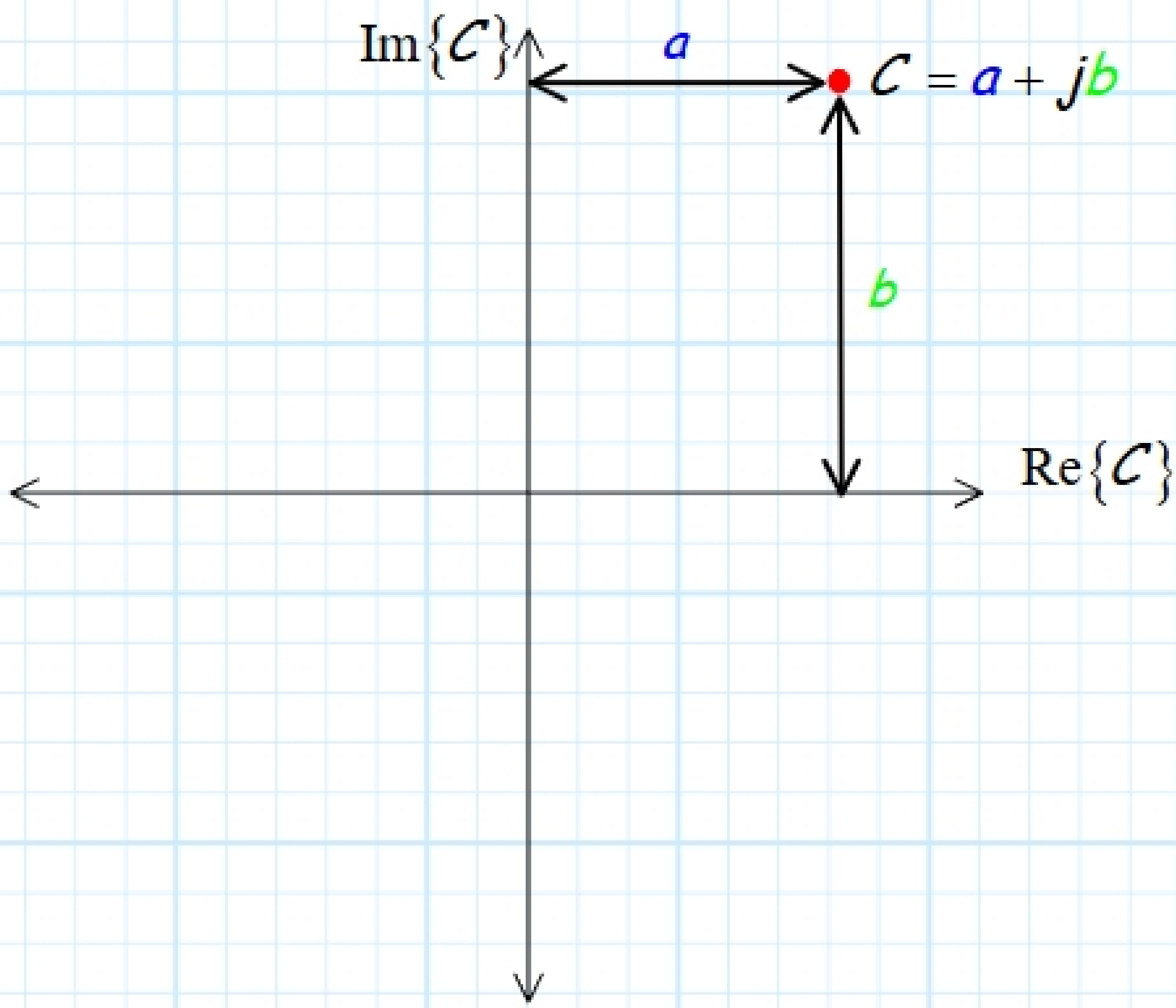
$$a = \mathrm{Re}\{C\} \qquad \text{and} \qquad b = \mathrm{Im}\{C\}$$

so that we can express this complex value as:

$$C = a + jb$$

where $j^2 = -1$.

Just as a **real** value can be expressed as a point on the **real line**, a **complex** value can be expressed as a point on the **complex plane**:

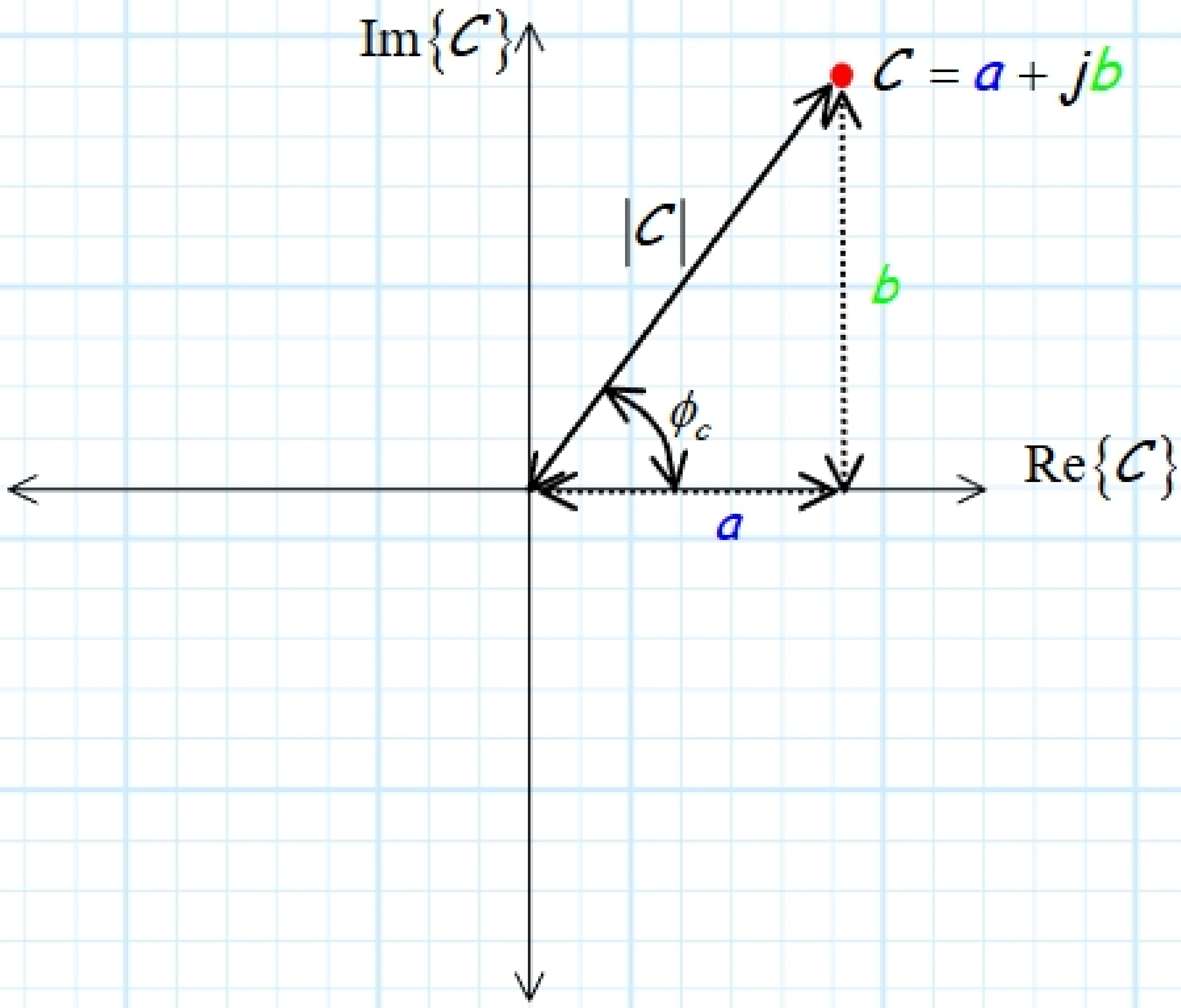


The values (a,b) are a **Cartesian** representation of a point on the complex plane. Recall that we can **alternatively** denote a point on a 2-dimensional plane using **polar** coordinates:

$|C| \doteq$ distance from the origin to the point

$\angle C \doteq \phi_c =$ rotation angle from the horizontal ($\text{Re}\{C\}$) axis

i.e.,



Using our knowledge of **trigonometry**, we can determine the relationship between the Cartesian (a,b) and polar ($|C|,\phi_c$) representations.

From the **Pythagorean** theorem, we find that:

$$|C| = \sqrt{a^2 + b^2}$$

Likewise, from the definition of ***sine*** (opposite over hypotenuse), we find:

$$\sin\phi_c = \frac{b}{|C|} = \frac{b}{\sqrt{a^2+b^2}}$$

or, using the definition of ***cosine*** (adjacent over hypotenuse):

$$\cos\phi_c = \frac{a}{|C|} = \frac{a}{\sqrt{a^2+b^2}}$$

Combining these results, we can determine the ***tangent*** (opposite over adjacent) of ϕ_c:

$$\tan\phi_c = \frac{\sin\phi_c}{\cos\phi_c} = \frac{b}{a}$$

Thus, we can write the polar coordinates **in terms of** the Cartesian coordinates:

$$|C| = \sqrt{a^2+b^2}$$

$$\phi_c = \tan^{-1}\left(b/a\right) = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right) = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$$