Fourier Analysis and Synthesis of Complex Waves

Introduction

In this lab we will study some aspects of digital synthesis of wave forms and Fourier analysis of waves to extract their frequency components. We will touch on the questions of noise spectra and analyze the noise spectrum produced by a zener diode.

Digital Synthesis

In this part of the lab we will synthesize periodic waveforms by the addition of sine waves. We will use the LabView program "Arbitrary Waveform Generator.vi" to build up waveforms in one of its windows by adding components assembled in the other. When the final assembly is complete on Channel 0, it will be converted to a continuous stream of pulses to be observed by an oscilloscope or alternately to be heard through earphones.

The frequency of the output wave is determined by setting the number of waveforms in the construction windows which contains 1,000 points and by specifying the number of points to read each second.

Wave Generation

The Sine Wave

Use the library feature of the program to set up a sine wave with amplitude 6 V, phase angle of 90°, 2 waveforms in the window and an acquisition rate of 100,000 points/second. What is the frequency of the resulting wave? Verify your conclusion by making a measurement with the oscilloscope. Also listen to the tone on the earphones.

The Triangle

Make an approximate 200 Hz triangle by adding a third harmonic to the sine above with amplitude chosen from the Fourier series expansion for a triangle:

$$x(t) = \frac{8}{\pi^2} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n^2} \cos(2\pi nt / T)$$

Add five more harmonics and observe the improvement of the triangle shape.

The Square Wave

Make an approximation to a 200 Hz square wave by using two components, the first and third harmonics, as given in the series below:

$$x(t) = \frac{4}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin(2\pi nt / T)$$

Observe the oscillations. Now add eight more terms one at a time. Observe that the oscillations do not go away but tend to be concentrated in the region of the discontinuity. In fact, with a finite number of terms, the oscillations at the discontinuity never go away. This effect is known as the "Gibbs phenomenon".

Crest factor

The shape of the waveform of a tone depends on the amplitude and the phases of the components. The power spectrum depends only on the amplitudes. Therefore, one can change the waveform shape while leaving the power spectrum the same by changing the phases of the components.

The crest factor is the maximum value of the waveform, divided by the RMS value. In communications practice, there is an advantage to keeping the crest factor low.

Below we consider a waveform having the first three harmonics, all with the same amplitude.

Show that the largest possible crest factor is obtained by choosing phases so as to add up cosine waves.

Show that the crest factor for three cosines of equal amplitude is $\sqrt{6}$.

Generate this wave. Observe it and listen to it.

The smallest crest factor can be obtained by reversing the sign of the third harmonic. (It is not obvious why should be so, but it is so.) Generate this wave. Compare its shape and its sound with the wave from part (c)

Components above the Nyquist frequency and fold-over distortion

Digital Synthesis creates components with high frequencies that are not desired in the output. The purpose of a reconstruction filter is to remove them. The generation of a sine tone is the simplest illustration. Suppose we want to generate a 5,000-Hz sine, using the 20,000 sample rate. In fact we generate quite a complex spectrum. Not only do we get 5,000 Hz. We get 20,000 ± 5,000. We also get (2 * 20,000) ± 5,000, and so on.

Explain why this means that the reconstruction filter ought to cutoff below a frequency, which is half the sample rate. Half the sample rate is a frequency known as the "Nyquist frequency".

Analog to digital conversion and the FFT

A LabView program, "Acquire&FFT_Nscans.vi", is available on your pc which (1) captures a waveform and digitizes it using an analog-to-digital converter (ADC), (2) takes the fast Fourier transform (FFT) of a specified number samples of the waveform, and (3) displays the Fourier transform on the monitor. For the number of samples always use a number that is a power of 2, i.e. 2^n, because the FFT program works much more efficiently on such a sample.

Digitize low-frequency waves from the function generator, sine, triangle, and square. Observe the power spectra on the monitor and compare with expected power spectra. (Note: Because there is no anti-aliasing filter in front of the ADC, some components of the triangle and square are bound to be above the Nyquist frequency (component frequencies greater than half the sample rate) and lead to aliasing. But if the fundamental frequency is low, the components should be small.)

Digitize a sine wave with frequency f_0 greater than half the sample rate f_{SR} Compare the frequency of the FFT peak with the calculated frequency of the alias, $f_{SR} - f_0$.

Noise

The best way to build a noise generator is to use a zener diode, reverse biased, and a high-gain amplifier. Figure 1 below shows how.

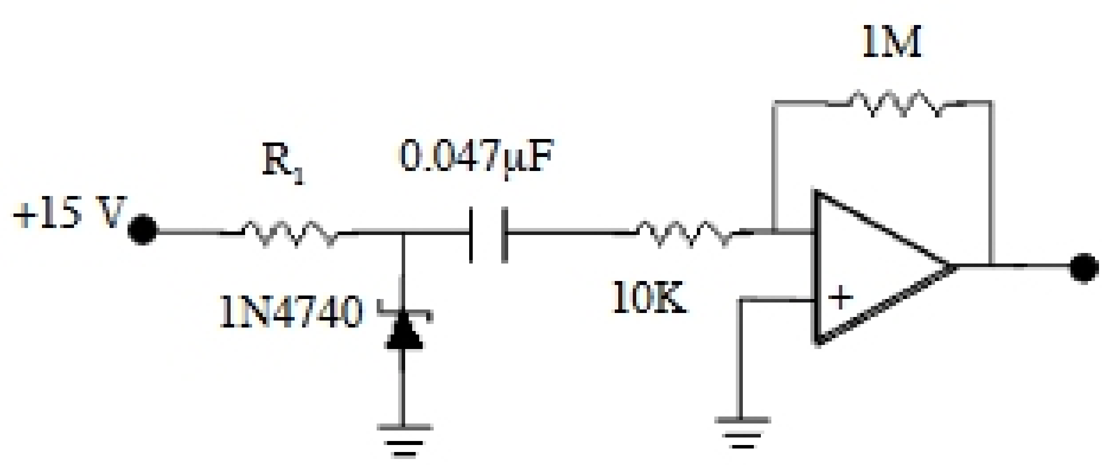

Figure 1: Noise Generation Circuit.

The strength and quality of the noise depends upon the diode and upon the current limiting resistor R_1. As a rule, high-voltage zeners work best. Because the power supply is 15 volts, a zener with breakdown voltage between 10 and 13 volts is recommended. Different zeners behave differently as noise sources. Even if they come from the same production lot they behave differently. The noise output from each zener is maximized by a different value of R_1. Some zeners don't make much noise for any value of R_1. Some zeners make a lot of noise but only for a rather specific value of R1. Other zeners make a lot of noise for a wide range of values of R_1. In the end, if you want to make a noise source, buy ten high-voltage zeners and select the best one.